LESSON

6-2 Parallelograms

:: Then

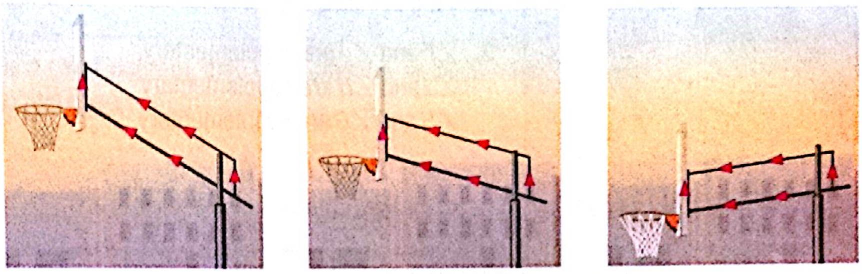
- You classified polygons with four sides as quadrilaterals. (Lesson 1-6)

:: Now

1. Recognize and apply properties of the sides and angles of parallelograms.
2. Recognize and apply properties of the diagonals of parallelograms.

:: Why?

- The arm of the basketball goal shown can be adjusted to a height of 10 feet or 5 feet. Notice that as the height is adjusted, each pair of opposite sides of the quadrilateral formed by the arms remains parallel.



NewVocabulary
parallelogram

Tennessee Curriculum Standards

✓ 3108.3.2 Connect coordinate geometry to geometric figures in the plane.

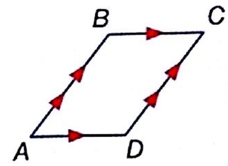
SPI 3108.3.2 Use coordinate geometry to prove characteristics of polygonal figures.

✓ 3108.4.10 Identify and apply properties and relationships of special figures.

1 Sides and Angles of Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. To name a parallelogram, use the symbol ▱. In ▱$ABCD$, $\overline{BC} \parallel \overline{AD}$ and $\overline{AB} \parallel \overline{DC}$ by definition.

Other properties of parallelograms are given in the theorems below.



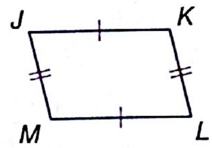
▱ABCD

Theorem Properties of Parallelograms

6.3 If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Abbreviation *Opp. sides of a ▱ are ≅.*

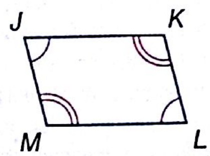
Example If $JKLM$ is a parallelogram, then $\overline{JK} \cong \overline{ML}$ and $\overline{JM} \cong \overline{KL}$.



6.4 If a quadrilateral is a parallelogram, then its opposite angles are congruent.

Abbreviation *Opp. ∠s of a ▱ are ≅.*

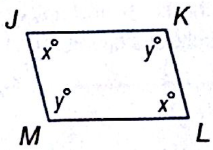
Example If $JKLM$ is a parallelogram, then $\angle J \cong \angle L$ and $\angle K \cong \angle M$.



6.5 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

Abbreviation *Cons. ∠s in a ▱ are supplementary.*

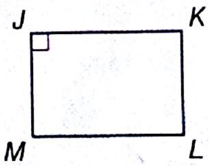
Example If $JKLM$ is a parallelogram, then $x + y = 180$.



6.6 If a parallelogram has one right angle, then it has four right angles.

Abbreviation *If a ▱ has 1 rt. ∠, it has 4 rt. ∠s.*

Example In ▱$JKLM$, if $\angle J$ is a right angle, then $\angle K$, $\angle L$, and $\angle M$ are also right angles.



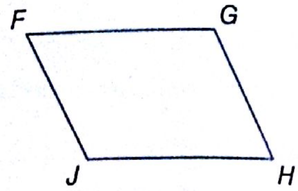
You will prove Theorems 6.3, 6.5, and 6.6 in Exercises 28, 26, and 7, respectively.

Study Tip

Including a Figure Theorems are presented in general terms. In a proof, you must include a drawing so that you can refer to segments and angles specifically.

Proof Theorem 6.4

Write a two-column proof of Theorem 6.4.

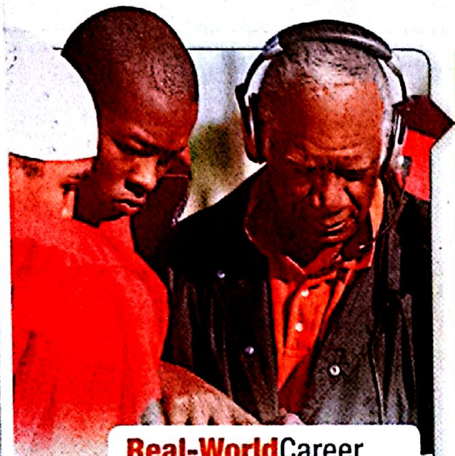


Given: ▱$FGHJ$

Prove: $\angle F \cong \angle H$, $\angle J \cong \angle G$

Proof:

Statements	Reasons
1. ▱$FGHJ$	1. Given
2. $\overline{FG} \parallel \overline{JH}$; $\overline{FJ} \parallel \overline{GH}$	2. Definition of parallelogram
3. $\angle F$ and $\angle J$ are supplementary. $\angle J$ and $\angle H$ are supplementary. $\angle H$ and $\angle G$ are supplementary.	3. If parallel lines are cut by a transversal, consecutive interior angles are supplementary.
4. $\angle F \cong \angle H$, $\angle J \cong \angle G$	4. Supplements of the same angles are congruent.



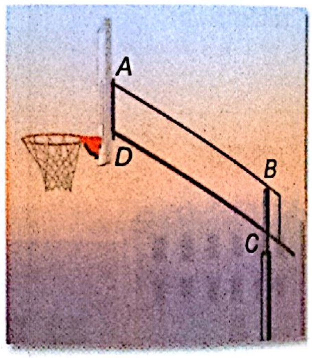
Real-World Career

Coach Coaches organize amateur and professional atheletes, teaching them the fundamentals of a sport. They manage teams during both practice sessions and competitions. Additional tasks may include selecting and issuing sports equiment, materials, and supplies. Head coaches at public secondary schools usually have a bachelor's degree.

Real-World Example 1 Use Properties of Parallelograms

PT

BASKETBALL In ▱$ABCD$, suppose $m\angle A = 55$, $AB = 2.5$ feet, and $BC = 1$ foot. Find each measure.



a. DC

$DC = AB$ — Opp. sides of a ▱ are ≅.

$= 2.5$ ft — Substitution

b. $m\angle B$

$m\angle B + m\angle A = 180$ — Cons. ∠s in a ▱ are supplementary.

$m\angle B + 55 = 180$ — Substitution

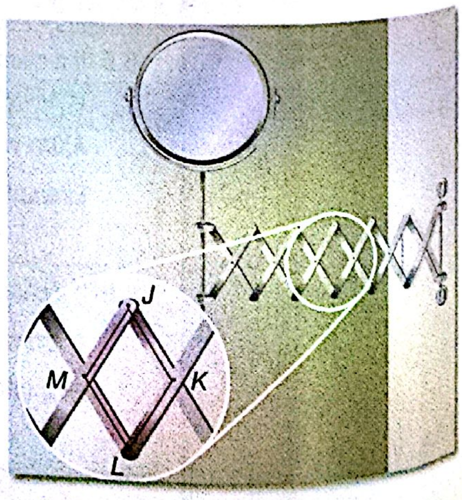
$m\angle B = 125$ — Subtract 55 from each side.

c. $m\angle C$

$m\angle C = m\angle A$ — Opp. ∠s of a ▱ are ≅.

$= 55$ — Substitution

Guided Practice



1. **MIRRORS** The wall-mounted mirror shown uses parallelograms that change shape as the arm is extended. In ▱$JKLM$, suppose $m\angle J = 47$. Find each measure.

   A. $m\angle L$ **B.** $m\angle M$

   C. Suppose the arm was extended further so that $m\angle J = 90$. What would be the measure of each of the other angles? Justify your answer.

2 Diagonals of Parallelograms

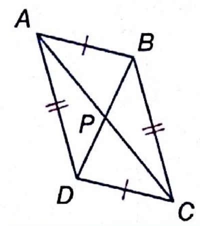
Diagonals of Parallelograms The diagonals of a parallelogram have special properties as well.

Theorem Diagonals of Parallelograms

6.7 If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Abbreviation Diag. of a ▱ bisect each other.

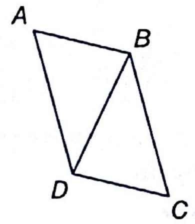
Example If *ABCD* is a parallelogram, then $\overline{AP} \cong \overline{PC}$ and $\overline{DP} \cong \overline{PB}$.



6.8 If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.

Abbreviation Diag. separates a ▱ into 2 ≅ △s.

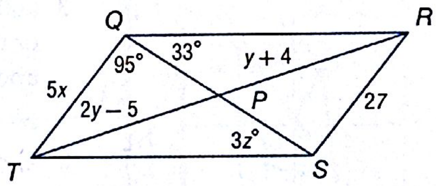
Example If *ABCD* is a parallelogram, then $\triangle ABD \cong \triangle CDB$.



You will prove Theorems 6.7 and 6.8 in Exercises 29 and 27, respectively.

Example 2 Use Properties of Parallelograms and Algebra

ALGEBRA If *QRST* is a parallelogram, find the value of the indicated variable.



a. x

$\overline{QT} \cong \overline{RS}$	Opp. sides of a ▱ are ≅.
$QT = RS$	Definition of congruence
$5x = 27$	Substitution
$x = 5.4$	Divide each side by 5.

b. y

$\overline{TP} \cong \overline{PR}$	Diag. of a ▱ bisect each other.
$TP = PR$	Definition of congruence
$2y - 5 = y + 4$	Substitution
$y = 9$	Subtract y and add 5 to each side.

c. z

$\triangle TQS \cong \triangle RSQ$	Diag. separates a ▱ into 2 ≅ △s.
$\angle QST \cong \angle SQR$	CPCTC
$m\angle QST = m\angle SQR$	Definition of congruence
$3z = 33$	Substitution
$z = 11$	Divide each side by 3.

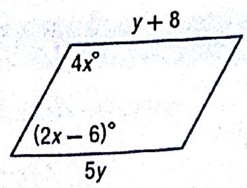
StudyTip

Congruent Triangles A parallelogram with two diagonals divides the figure into two pairs of congruent triangles.

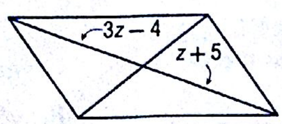
GuidedPractice

Find the value of each variable in the given parallelogram.

2A.



2B.



You can use Theorem 6.7 to determine the coordinates of the intersection of the diagonals of a parallelogram on a coordinate plane given the coordinates of the vertices.

Example 3 Parallelograms and Coordinate Geometry

COORDINATE GEOMETRY Determine the coordinates of the intersection of the diagonals of ▱*FGHJ* with vertices *F*(−2, 4), *G*(3, 5), *H*(2, −3), and *J*(−3, −4).

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{FH}$ and $\overline{GJ}$. Find the midpoint of $\overline{FH}$ with endpoints (−2, 4) and (2, −3).

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 2}{2}, \frac{4 + (-3)}{2}\right) \quad \text{Midpoint Formula}$$

$$= (0, 0.5) \quad \text{Simplify.}$$

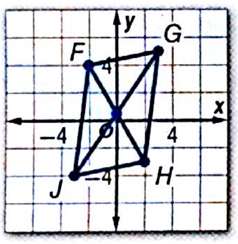
The coordinates of the intersection of the diagonals of ▱*FGHJ* are (0, 0.5).

CHECK Find the midpoint of $\overline{GJ}$ with endpoints (3, 5) and (−3, −4).

$$\left(\frac{3 + (-3)}{2}, \frac{5 + (-4)}{2}\right) = (0, 0.5) \checkmark$$

StudyTip

Check for Reasonableness Graph the parallelogram in Example 3 and the point of intersection of the diagonals you found. Draw the diagonals. The point of intersection appears to be correct.



GuidedPractice

3. **COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of *RSTU* with vertices *R*(−8, −2), *S*(−6, 7), *T*(6, 7), and *U*(4, −2).

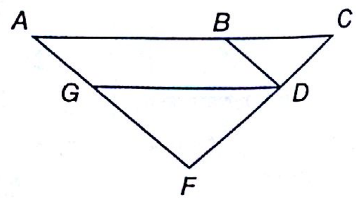
You can use the properties of parallelograms and their diagonals to write proofs.

Example 4 Proofs Using the Properties of Parallelograms

Write a paragraph proof.

Given: ▱*ABDG*, $\overline{AF} \cong \overline{CF}$

Prove: ∠*BDG* ≅ ∠*C*



Proof:

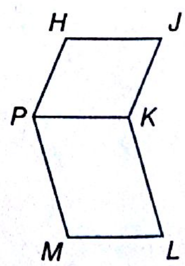
We are given *ABDG* is a parallelogram. Since opposite angles in a parallelogram are congruent, ∠*BDG* ≅ ∠*A*. We are also given that $\overline{AF} \cong \overline{CF}$. By the Isosceles Triangle Theorem, ∠*A* ≅ ∠*C*. So, by the Transitive Property of Congruence, ∠*BDG* ≅ ∠*C*.

GuidedPractice

4. Write a two-column proof.

   Given: ▱*HJKP* and ▱*PKLM*

   Prove: $\overline{HJ} \cong \overline{ML}$

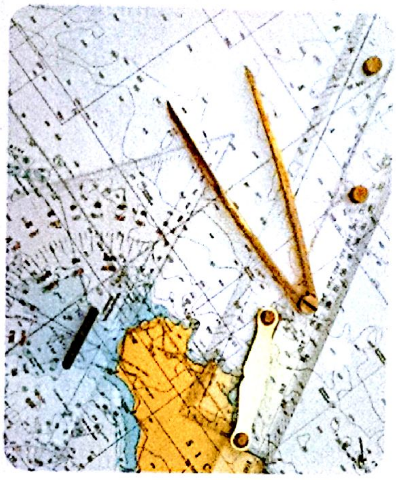


Check Your Understanding

= Step-by-Step Solutions begin on page R20.

Example 1

1. **NAVIGATION** To chart a course, sailors use a *parallel ruler*. One edge of the ruler is placed along the line representing the direction of the course to be taken. Then the other ruler is moved until its edge reaches the compass rose printed on the chart. Reading the compass determines which direction to travel. The rulers and the crossbars form of the tool ▱$MNPQ$.



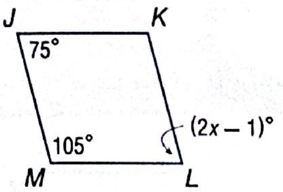
   a. If $m\angle NMQ = 32$, find $m\angle MNP$.

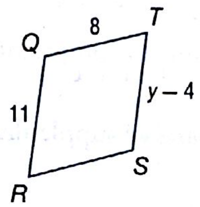
   b. If $m\angle MQP = 125$, find $m\angle MNP$.

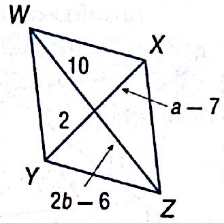
   c. If $MQ = 4$, what is NP?

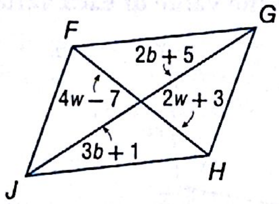
Example 2

ALGEBRA **Find the value of each variable in each parallelogram.**

2.


3.


4.


5.


Example 3

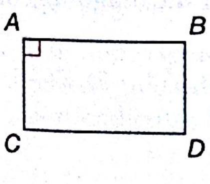
6. **COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of ▱$ABCD$ with vertices $A(-4, 6)$, $B(5, 6)$, $C(4, -2)$, and $D(-5, -2)$.

Example 4

PROOF **Write the indicated type of proof.**

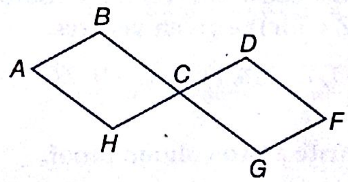
7. paragraph

   Given: ▱$ABCD$, $\angle A$ is a right angle.
   Prove: $\angle B$, $\angle C$, and $\angle D$ are right angles. (Theorem 6.6)



8. two-column

   Given: $ABCH$ and $DCGF$ are parallelograms.
   Prove: $\angle A \cong \angle F$

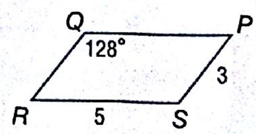


Practice and Problem Solving

Extra Practice begins on page 969.

Example 1

Use ▱$PQRS$ to find each measure.



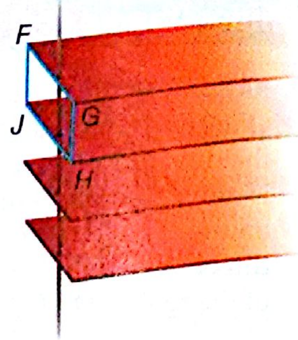
9. $m\angle R$

10. QR

11. QP

12. $m\angle S$

13. HOME DECOR The slats on Venetian blinds are designed to remain parallel in order to direct the path of light coming in a widow. In ▱$FGHJ$, $FJ = \frac{3}{4}$ inch, $FG = 1$ inch, and $\angle JHG = 62$. Find each measure.



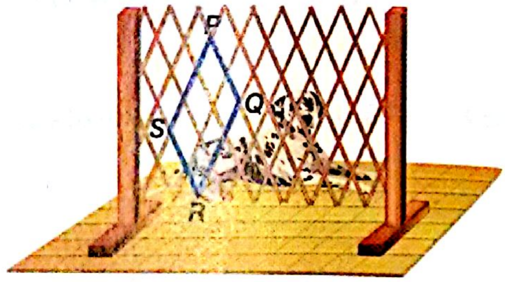
a. JH

b. GH

c. $m\angle JFG$

d. $m\angle FJH$

14. DOG SHOWS Wesley is a member of the kennel club in his area. His club uses accordion fencing like the section shown at the right to block out areas at dog shows.



a. Identify two pairs of congruent segments.

b. Identify two pairs of supplementary angles.

Example 2 **ALGEBRA** **Find the value of each variable in each parallelogram.**

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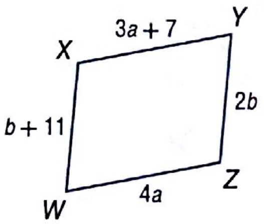

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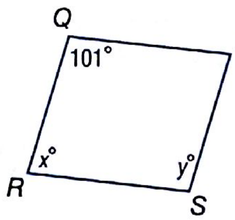

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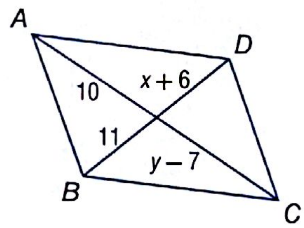

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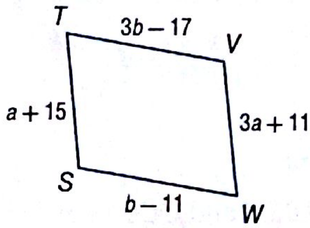

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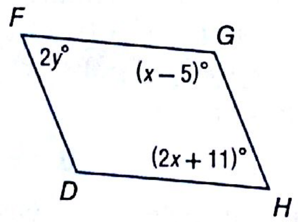

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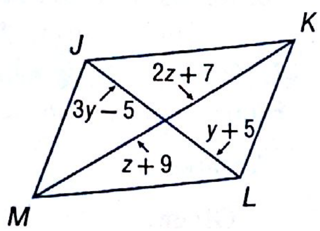


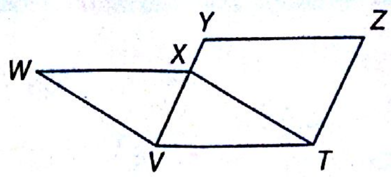
Example 3 **COORDINATE GEOMETRY** **Find the coordinates of the intersection of the diagonals of ▱$WXYZ$ with the given vertices.**

21. $W(-1, 7), X(8, 7), Y(6, -2), Z(-3, -2)$

22. $W(-4, 5), X(5, 7), Y(4, -2), Z(-5, -4)$

Example 4 **PROOF** **Write a two-column proof.**

23. **Given:** $WXTV$ and $ZYVT$ are parallelograms.
Prove: $\overline{WX} \cong \overline{ZY}$



24. **Given:** ▱$BDHA$, $\overline{CA} \cong \overline{CG}$
Prove: $\angle BDH \cong \angle G$