Proofs

Prove it!

Saying "proofs" to someone else is a sure fire to get their anxiety and blood pressure levels to start rising. Most adults fear math proofs more that they fear public speaking. But, it doesn't have to be that way if you can understand a few simple principles and take is slow. Here we go...

First, what is a proof? A <u>Proof</u> is merely a logical argument that demonstrates that some statement is in fact true.

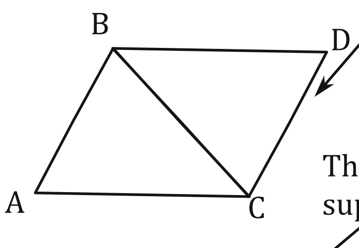
Here's a real life example. John Doe is on trial for stealing a Snickers bar from a convenience store. He is unfortunately caught in the act on tape. The prosecution presents the tape and shows the jury John Doe's face. This logical argument proves that the statement, "John Doe robbed the store" is true.

Okay, but what if we wanted to prove something like... the Pythagorean theorem? Considering the lack of camera technology during Euclid's life we will have to come up with something else.

We could, option A: Plug in every single possible value for all the sides of every right triangle that could possibly exist into this elegant equation and see if it is true. However, this would take literally forever considering there is an infinite number of possibilities. Even if we started doing this when Euclid was a baby (several thousand years ago) we would still not have barely even started by today. So... there is option B: Using accepted facts and properties of numbers and of logic to construct an argument that shows that this must be true.

I don't know about you, but I like the sound of B better. The Pythagorean theorem is a little tricky so let's start with something a bit easier. But, before we begin, let's talk about the format of a proof. First, a proof always starts with some given information. In the case of a Geometric proof this information usually includes a diagram and some other info. You will then be asked to prove some statement. There are two predominant forms from here. The first is called a two column proof which of course has two columns. One for the statements and the other for the theorem, postulate, or property that allows you to make the statement. The other is a paragraph proof. This has the same two elements, but is usually written in the form of a paragraph. We will focus on the former since it is what you will see most often. Let's look at a <u>Two Column Proof.</u>

Given: $\overline{AB}\cong\overline{DC}$ and $\overline{AC}\cong\overline{DB}$ ← This is stuff you can use.



This is what you are supposed to show is true.

Prove: $\triangle ABC\cong\triangle DBC$

Statements	Reasons
1. AB≅DC	1. Given
2. AC≅DB	2. Given
3. BC≅BC	3. Reflexive Property
4. $\triangle ABC\cong\triangle DBC$	4. SSS

These are the statements in the proof. Read them from top to bottom.

These are justifications for each statement to the left.

Notice that Line 4 could not be mentioned earlier because the three sides had to be listed first. In a proof you may use any statement from the given information, from the diagram or ANY OTHER PREVIOUS STATEMENT. This is critical. You can't say John Doe is guilty before presenting the facts.

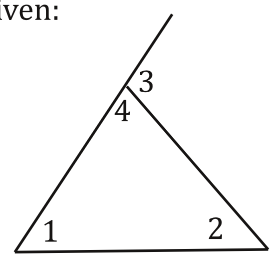
On that last example you might have noticed some strange things on the right like "Given" and "Reflexive Property". "SSS" you should remember as a triangle congruence theorem. If not, you should go back and review them before moving on. Here are what these things... and more mean.

Given. This appears on the right side of a proof and simply means you are stating information that was in the given information. It may also include things that are MARKED on the diagrams. (Stuff that isn't specifically marked, like the shared lines in the last example, can't be labeled "given.")

Reflexive Property. All this means, is that something is equal to itself. Like BC is exactly as long as BC is or even more simply x=x or 3=3 or m∠A=m∠A.

Here is another proof that has a couple more weird properties.

Given:



Statements	Reasons
1. m∠1+m∠2+m∠4=180°	1. Triangle Sum Theorem
2. m∠3+M∠4=180°	2. Linear Pair
3. m∠1+m∠2+m∠4=m∠3+m∠4	3. Reflexive Property
4. m∠1+m∠2=m∠3	4. Subtraction Property of Equality

Prove: m∠1+m∠2=m∠3

By the way, this proves the remote exterior angle theorem... How fun!

Substitution: This one should ring a bell from Algebra. It means if two things are equal to the same thing then they are equal . Like if John and Jane are both as tall as Tom, then they are the same height. Or If x=y and y=3 then x=3. This is pretty much the same thing as the law of syllogism! So, in the example ... m∠1+m∠2+m∠4=180° and m∠3+M∠4=180°, too. That means they are equal to each other... m∠1+m∠2+m∠4=m∠3+m∠4.

Addition Property of Equality, and the Multiplicative, Division, Subtraction, etc. Property of Equality. This is the basis of most of your fun in algebra class. If you add something to both sides of an equation, then both sides of the equation are still equal, right? Like x+3=6. Subtracting 3 from both sides solves the equation x=3. It also ensures the equality is still true. The additive and subtractive are the most commonly used in Geometric proofs. In the example, line 4.

Now you won't be asked to do a proof this complicated. At least not yet...

First you will be writing analyses of some proofs. Then you will be filling in the blanks. Finally, you will do a few on your own. We will start with Triangle Congruence, then add CPCTC, and then do some on Triangle Similarity.

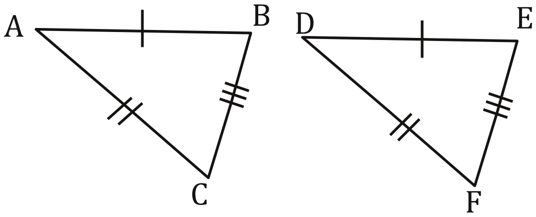
Before we get into doing proofs, let's look at a way to think about proofs that seems to work the best...

The Backwards/Forwards Method for Doing Proofs

How to make the beasties more tame.

If you read the section on solving more difficult problems called the "Backwards Method" this will be a piece of cake. To apply that method to proofs, do this. Start with the "prove" and work your way backwards until you get stuck...then go to the "Given" information and work forwards until you tie it all together. The advantage of doing it this way is that when you start looking at the "Given" information you have a target and a purpose for looking at it. Otherwise the reason why the "Given" information exists is quite confusing. Let's look at a simple proof to show you how to use it.

Given: $\overline{AB}\cong\overline{DE}$, $\overline{AC}\cong\overline{DF}$, and $\overline{BC}\cong\overline{EF}$



Prove: $\triangle ABC\cong\triangle DEF$

Statements	Reasons
F1: $\overline{AB}\cong\overline{DE}$	1. Given
F2: $\overline{AC}\cong\overline{DF}$	2. Given
F3. $\overline{BC}\cong\overline{EF}$	3. Given
B1. $\triangle ABC\cong\triangle DEF$	4. SSS

To use the method, start with the "Prove." In this case specifically "Prove $\Delta ABC\cong\Delta DBC$." This seems backwards because the part that says "Prove" is towards the bottom. When you read the "Prove" you see you are asked to prove or show that two triangles are congruent. Now... you probably have asked yourself. "How am I supposed to do that?" That is actually the first Key Question! "How can I prove two triangles are congruent?" The answer is of course which one? We have SSS, SAS, ASA, AAS and HL. That's okay, just write B1: (backwards step one) is $\Delta ABC\cong\Delta DEF$ by some property. Well, here you are likely stuck, because which one should you use? Now start using the Given information and work forward. Write F1: $\overline{AB}\cong\overline{DE}$ Given, then F2: $\overline{AC}\cong\overline{DF}$ Given, and then F3: $\overline{BC}\cong\overline{EF}$ Given. Okay, now take a look... Guess what you have, three pairs of congruent sides. This is SSS and you are at B1! You are so smart! Your proof is complete! That was easy, huh?

Notice that the numbering on a proof that you would actually do would be numbered from 1 to 4 going forward. The numbering on this one was just to demonstrate the method. It works so well because going forward is difficult when you are not sure where you are going. When you actually do proofs sometimes they will be a little more difficult. Study the analysis on the next page to get a feel for how to use the method on other types of proofs.

This method is not just useful for proofs, but really anywhere you have a multistep problem that looks confusing. If you start from the end and go backwards, it's a lot easier. In fact, most people who are good at these kinds of problems naturally, do this without knowing it.

I've told you the secret... Use it wisely.