

Centers of Triangles

Mathematical goals

- Prove the medians of a triangle meet at a point.
- Bisect a segment and an angle.
- Construct perpendicular lines, including the perpendicular bisector of a line segment.

Common Core State Standards

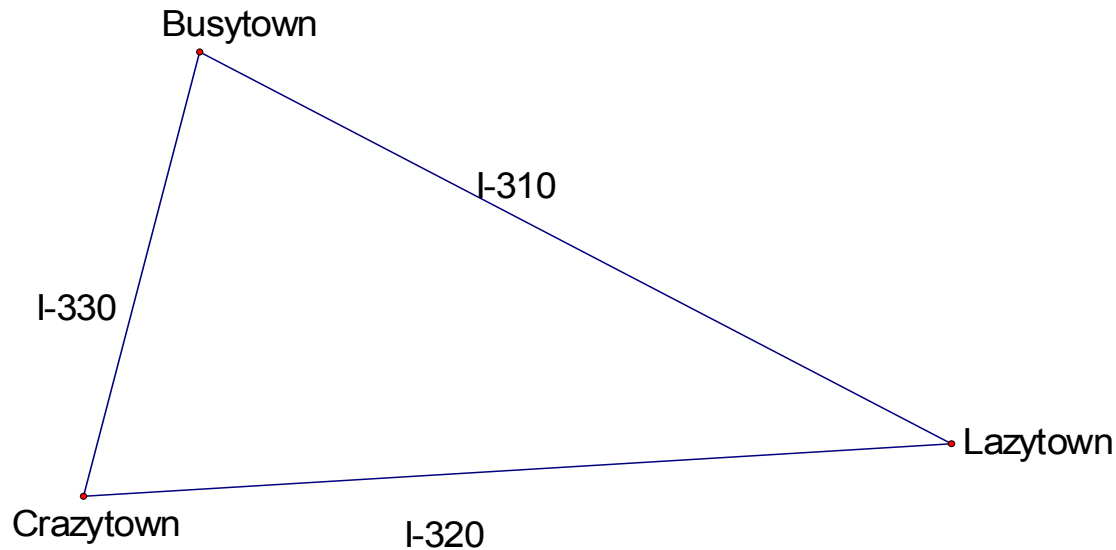
MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Triangle Center:	Point of Concurrency of:	Significance of:
Incenter	Angle bisectors	Center of inscribed circle Equidistant from the sides of the triangle
Circumcenter	Perpendicular bisectors	Center of the circumscribing circle Equidistant from the vertices of the
Orthocenter	Altitudes	
Centroid	Medians	Center of balance or gravity The distance from a vertex to the centroid is twice the distance from the centroid to the opposite side.

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or to the existing highways.



1. Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.
2. Now you will use some mathematical concepts to help you choose a location for the tower. Investigate the problem above by constructing the following:
 - a) all 3 medians of the triangle
 - b) all 3 altitudes of the triangle
 - c) all 3 angle bisectors of the triangle
 - d) all 3 perpendicular bisectors of the triangle

You have four different kinds of tools at your disposal- patty paper, MIRA, compass and straight edge, and Geometer's Sketch Pad. Use a different tool for each of your constructions.

3. Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point.
4. How close is the point you chose in part 3, based on mathematics, to the point you chose by observation?

You have now discovered that each set of segments resulting from the constructions above always has a point of intersection. These four points of intersection are called the ***points of concurrency*** of a triangle.

The intersection point of the medians is called the ***centroid*** of the triangle.

The intersection point of the angle bisectors is called the ***incenter*** of the triangle.

The intersection point of the perpendicular bisectors is called the ***circumcenter*** of the triangle.

The intersection point of the altitudes is called the ***orthocenter*** of the triangle.

5. Can you give a reasonable guess as to why the specific names were given to each point of concurrency?
6. Which triangle center did you recommend for the location of the amusement park?
7. The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation.