4 Study Guide and Review

Study Guide

Key Concepts

Classifying Triangles (Lesson 4-1)

- Triangles can be classified by their angles as acute, obtuse, or right, and by their sides as scalene, isosceles, or equilateral.

Angles of Triangles (Lesson 4-2)

- The measure of an exterior angle is equal to the sum of its two remote interior angles.

Congruent Triangles (Lesson 4-3 through 4-5)

- SSS: If all of the corresponding sides of two triangles are congruent, then the triangles are congruent.
- SAS: If two pairs of corresponding sides of two triangles and the included angles are congruent, then the triangles are congruent.
- ASA: If two pairs of corresponding angles of two triangles and the included sides are congruent, then the triangles are congruent.
- AAS: If two pairs of corresponding angles of two triangles are congruent, and a corresponding pair of nonincluded sides is congruent, then the triangles are congruent.

Isosceles and Equilateral Triangles (Lesson 4-6)

- The base angles of an isosceles triangle are congruent and a triangle is equilateral if it is equiangular.

Transformations and Coordinate Proofs (Lessons 4-7 and 4-8)

- In a congruence transformation, the position of the image may differ from the preimage, but the two figures remain congruent.
- Coordinate proofs use algebra to prove geometric concepts.

FOLDABLES Study Organizer

Be sure the Key Concepts are noted in your Foldable.

Key Vocabulary

acute triangle (p. 235)
auxiliary line (p. 244)
base angles (p. 283)
congruence transformation (p. 294)
congruent polygons (p. 253)
coordinate proof (p. 301)
corollary (p. 247)
corresponding parts (p. 253)
equiangular triangle (p. 235)
equilateral triangle (p. 236)
exterior angle (p. 246)
flow proof (p. 246)
included angle (p. 264)
included side (p. 273)
isosceles triangle (p. 236)
obtuse triangle (p. 235)
reflection (p. 294)
remote interior angles (p. 246)
right triangle (p. 235)
rotation (p. 294)
scalene triangle (p. 236)
translation (p. 294)
vertex angle (p. 283)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. An equiangular triangle is also an example of an acute triangle.
2. A triangle with an angle that measures greater than 90° is a right triangle.
3. An equilateral triangle is always equiangular.
4. A scalene triangle has at least two congruent sides.
5. The vertex angles of an isosceles triangle are congruent.
6. An included side is the side located between two consecutive angles of a polygon.
7. The three types of congruence transformations are rotation, reflection, and translation.
8. A rotation moves all points of a figure the same distance and in the same direction.
9. A flow proof uses figures in the coordinate plane and algebra to prove geometric concepts.
10. The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Lesson-by-Lesson Review

✓3108.4.9, SPI 3108.4.2

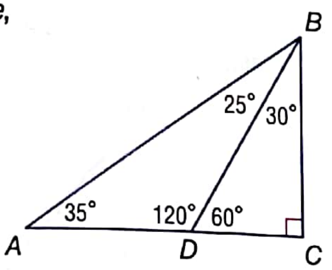
4-1 Classifying Triangles (pp. 235–242)

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

11. $\triangle ADB$

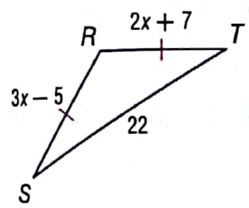
12. $\triangle BCD$

13. $\triangle ABC$



ALGEBRA Find x and the measures of the unknown sides of each triangle.

14.



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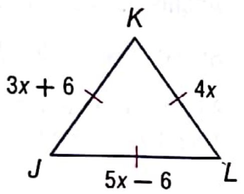
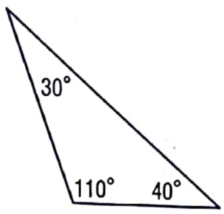


16. **MAPS** The distance from Chicago to Cleveland to Cincinnati and back to Chicago is 900 miles. The distance from Chicago to Cleveland is 50 miles more than the distance from Cincinnati to Chicago, and the distance from Cleveland to Cincinnati is 50 miles less than the distance from Cincinnati to Chicago. Find each distance and classify the triangle formed by the three cities.

Example 1

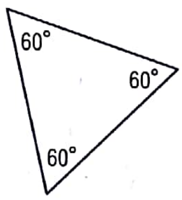
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

a.



Since the triangle has one obtuse angle, it is an obtuse triangle.

b.



The triangle has three acute angles that are all equal. It is an equiangular triangle.

SPI 3108.1.4, CLE 3108.4.3, SPI 3108.4.4

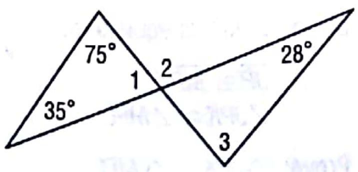
4-2 Angles of Triangles (pp. 244–252)

Find the measure of each numbered angle.

17. $\angle 1$

18. $\angle 2$

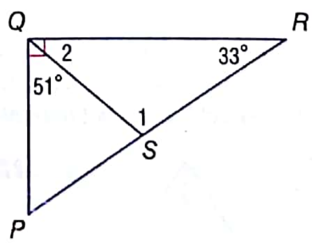
19. $\angle 3$



20. **HOUSES** The roof support on Lamar's house is in the shape of an isosceles triangle with base angles of 38°. Find x.

Example 2

Find the measure of each numbered angle.



$m\angle 2 + m\angle PQS = 90$	
$m\angle 2 + 51 = 90$	Substitution
$m\angle 2 = 39$	Subtract 51 from each side.
$m\angle 1 + m\angle 2 + 33 = 180$	Triangle Sum Theorem
$m\angle 1 + 39 + 33 = 180$	Substitution
$m\angle 1 + 72 = 180$	Simplify.
$m\angle 1 = 108$	Subtract.

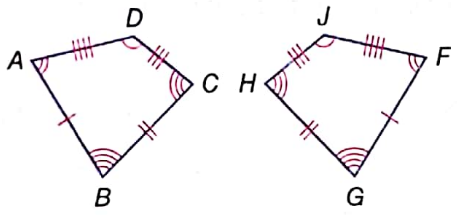
4 Study Guide and Review *Continued*

CLE 3108.4.8, ✓3108.4.38, SPI 3108.4.11

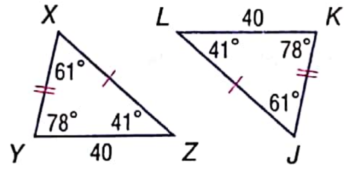
4-3 Congruent Triangles (pp. 253–261)

Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

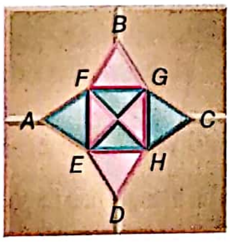
21.



22.

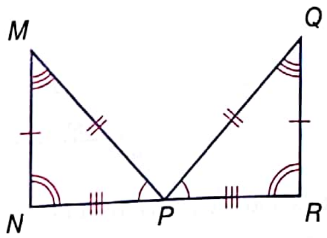


23. MOSAIC TILING A section of a mosaic tiling is shown. Name the triangles that appear to be congruent.



Example 3

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.



Angles: $\angle N \cong \angle R$, $\angle M \cong \angle Q$, $\angle MPN \cong \angle QPR$

Sides: $\overline{MN} \cong \overline{QR}$, $\overline{MP} \cong \overline{QP}$, $\overline{NP} \cong \overline{RP}$

All corresponding parts of the two triangles are congruent. Therefore, $\triangle MNP \cong \triangle QRP$.

CLE 3108.4.8, ✓3108.4.35

4-4 Proving Triangles Congruent—SSS, SAS (pp. 262–270)

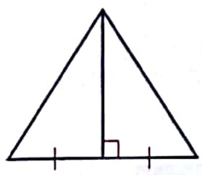
Determine whether $\triangle ABC \cong \triangle XYZ$. Explain.

24. $A(5, 2)$, $B(1, 5)$, $C(0, 0)$, $X(-3, 3)$, $Y(-7, 6)$, $Z(-8, 1)$

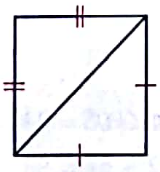
25. $A(3, -1)$, $B(3, 7)$, $C(7, 7)$, $X(-7, 0)$, $Y(-7, 4)$, $Z(1, 4)$

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

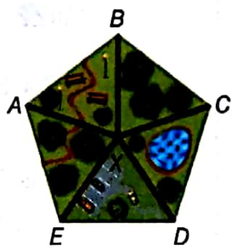
26.



27.



28. PARKS The diagram shows a park in the shape of a pentagon with five sidewalks of equal length leading to a central point. If all the angles at the central point have the same measure, how could you prove that $\triangle ABX \cong \triangle DCX$?

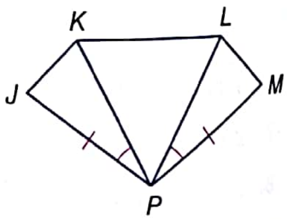


Example 4

Write a two-column proof.

Given: $\triangle KPL$ is equilateral.
$\overline{JP} \cong \overline{MP}$,
$\angle JPK \cong \angle MPL$

Prove: $\triangle JPK \cong \triangle MPL$



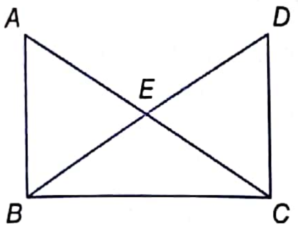
Statements	Reasons
1. $\triangle KPL$ is equilateral.	1. Given
2. $\overline{PK} \cong \overline{PL}$	2. Def. of Equilateral △
3. $\overline{JP} \cong \overline{MP}$	3. Given
4. $\angle JPK \cong \angle MPL$	4. Given
5. $\triangle JPK \cong \triangle MPL$	5. SAS

4-5 Proving Triangles Congruent—ASA, AAS (pp. 273–280)

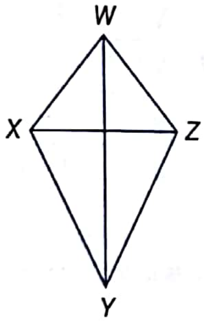
Write a two-column proof.

29. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABE \cong \triangle CDE$



30. KITES Denise's kite is shown in the figure at the right. Given that $\overline{WY}$ bisects both $\angle XWZ$ and $\angle XYZ$, prove that $\triangle WXY \cong \triangle WZY$.

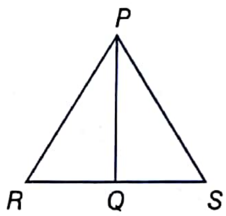


Example 5

Write a flow proof.

Given: $\overline{PQ}$ bisects $\angle RPS$.
$\angle R \cong \angle S$

Prove: $\triangle RPQ \cong \triangle SPQ$



Flow Proof:

$\overline{PQ}$ bisects $\angle RPS$. — **Given** → $\angle RPQ \cong \angle SPQ$ — Def. of $\angle$ Bisector

$\angle R \cong \angle S$ — **Given**

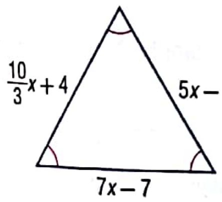
$\overline{PQ} \cong \overline{PQ}$ — **Reflexive**

→ $\triangle RPQ \cong \triangle SPQ$ — AAS

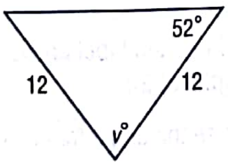
4-6 Isosceles and Equilateral Triangles (pp. 283–291)

Find the value of each variable.

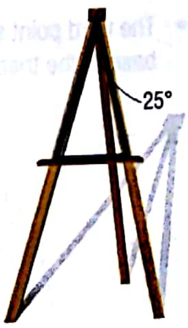
31.



32.

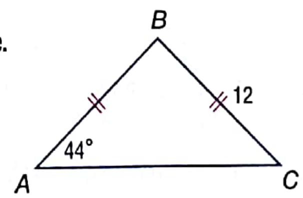


33. PAINTING Pam is painting using a wooden easel. The support bar on the easel forms an isosceles triangle with the two front supports. According to the figure below, what are the measures of the base angles of the triangle?



Example 6

Find each measure.



a. $m\angle B$

Since $AB = BC$, $\overline{AB} \cong \overline{BC}$. By the Isosceles Triangle Theorem, base angles A and C are congruent, so $m\angle A = m\angle C$. Use the Triangle Sum Theorem to write and solve an equation to find $m\angle B$.

$m\angle A + m\angle B + m\angle C = 180$	$\triangle$ Sum Theorem
$44 + m\angle B + 44 = 180$	$m\angle A = m\angle C = 44$
$88 + m\angle B = 180$	Simplify.
$m\angle B = 92$	Subtract.

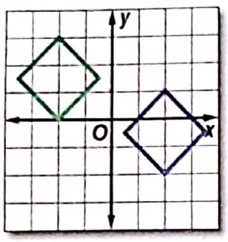
b. AB

$AB = BC$, so $\triangle ABC$ is isosceles. Since $BC = 12$, $AB = 12$ by substitution.

CLE 3108.4.8, ✓3108.4.31, ✓3108.4.34

4-7 Congruence Transformations (pp. 294–300)

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

34.


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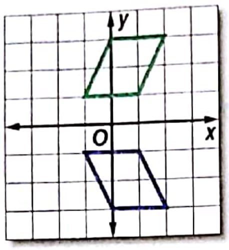
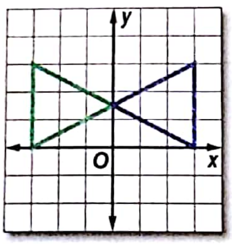
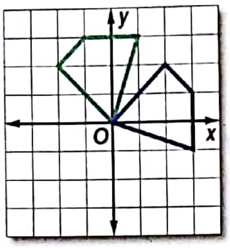


36.


37.


38. Triangle ABC with vertices $A(1, 1)$, $B(2, 3)$, and $C(3, -1)$ is a transformation of △MNO with vertices $M(-1, 1)$, $N(-2, 3)$, and $O(-3, -1)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Example 7

Triangle *RST* with vertices $R(4, 1)$, $S(2, 5)$, and $T(-1, 0)$ is a transformation of $\triangle CDF$ with vertices $C(1, -3)$, $D(-1, 1)$, and $F(-4, -4)$. Identify the transformation and verify that it is a congruence transformation.

Graph each figure. The transformation appears to be a translation. Find the lengths of the sides of each triangle.

$RS = \sqrt{(4-2)^2 + (1-5)^2}$ or $\sqrt{20}$

$TS = \sqrt{(-1-2)^2 + (0-5)^2}$ or $\sqrt{34}$

$RT = \sqrt{(-1-4)^2 + (0-1)^2}$ or $\sqrt{26}$

$CD = \sqrt{(-1-1)^2 + [1-(-3)]^2}$ or $\sqrt{20}$

$DF = \sqrt{[-4-(-1)]^2 + (-4-1)^2}$ or $\sqrt{34}$

$CF = \sqrt{(-4-1)^2 + [-4-(-3)]^2}$ or $\sqrt{26}$

Since each vertex of $\triangle CDF$ has undergone a transformation 3 units to the right and 4 units up, this is a translation.

Since $RS = CD$, $TS = DF$, and $RT = CF$, $\overline{RS} \cong \overline{CD}$, $\overline{TS} \cong \overline{DF}$, and $\overline{RT} \cong \overline{CF}$. By SSS, $\triangle RST \cong \triangle CDF$.

CLE 3108.3.1, SPI 3108.3.2, CLE 3108.4.3

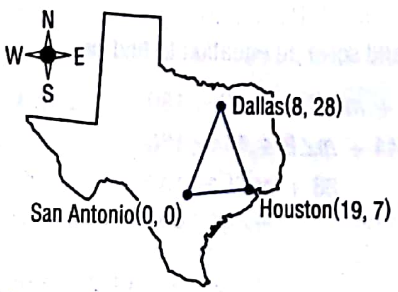
4-8 Triangles and Coordinate Proof (pp. 301–307)

Position and label each triangle on the coordinate plane.

39. right $\triangle MNO$ with right angle at point *M* and legs of lengths *a* and 2*a*.

40. isosceles $\triangle WXY$ with height *h* and base $\overline{WY}$ with length 2*a*.

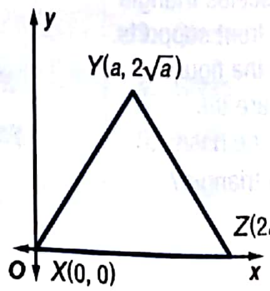
41. GEOGRAPHY Jorge plotted the cities of Dallas, San Antonio, and Houston as shown. Write a coordinate proof to show that the triangle formed by these cities is scalene.



Example 8

Position and label an equilateral triangle $\triangle XYZ$ with side lengths of 2*a*.

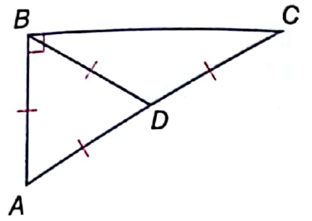
- Use the origin for one of the three vertices of the triangle.
- Place one side of the triangle along the positive side of the *x*-axis.
- The third point should be located above the midpoint of the base of the triangle.



CHAPTER 4

Practice Test

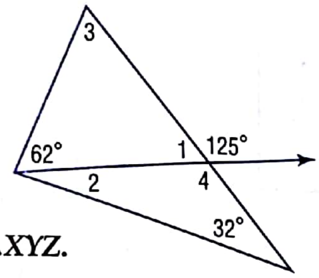
Classify each triangle as *acute*, *equilateral*, *obtuse*, or *right*.



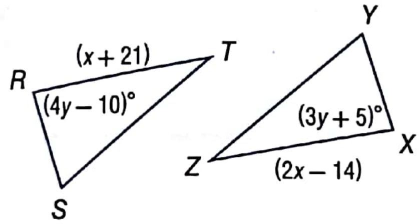
1. $\triangle ABD$
2. $\triangle ABC$
3. $\triangle BDC$

Find the measure of each numbered angle.

4. $\angle 1$
5. $\angle 2$
6. $\angle 3$
7. $\angle 4$



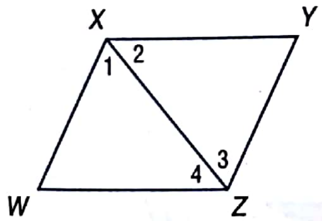
In the diagram, $\triangle RST \cong \triangle XYZ$.



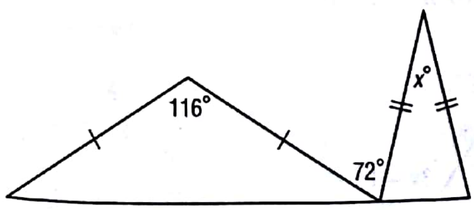
8. Find x.
9. Find y.
10. **PROOF** Write a flow proof.

    Given: $\overline{XY} \parallel \overline{WZ}$ and $\overline{XW} \parallel \overline{YZ}$

    Prove: $\triangle XWZ \cong \triangle ZYX$



11. **MULTIPLE CHOICE** Find x.



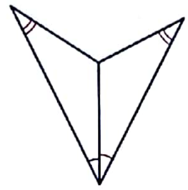
A 36   C 28

B 32   D 22

12. Determine whether $\triangle TJD \cong \triangle SEK$ given $T(-4, -2)$, $J(0, 5)$, $D(1, -1)$, $S(-1, 3)$, $E(3, 10)$, and $K(4, 4)$. Explain.

Determine which postulate or theorem can be used to prove each pair of triangles congruent. If it is not possible to prove them congruent, write *not possible*.

13.



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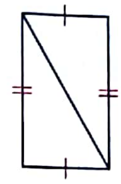

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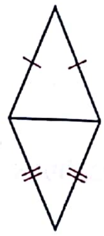

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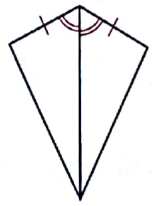
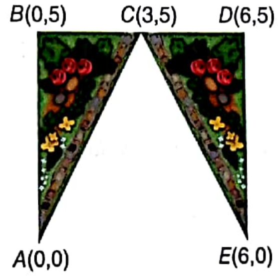
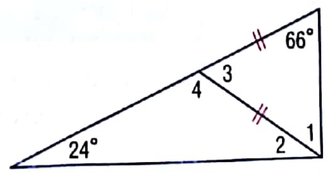


17. **LANDSCAPING** Angie has laid out a design for a garden consisting of two triangular areas as shown below. The points are $A(0, 0)$, $B(0, 5)$, $C(3, 5)$, $D(6, 5)$, and $E(6, 0)$. Name the type of congruence transformation for the preimage $\triangle ABC$ to $\triangle EDC$.



Find the measure of each numbered angle.

18. $\angle 1$
19. $\angle 2$



20. **PROOF** $\triangle ABC$ is a right isosceles triangle with hypotenuse $\overline{AB}$. M is the midpoint of $\overline{AB}$. Write a coordinate proof to show that $\overline{CM}$ is perpendicular to $\overline{AB}$.