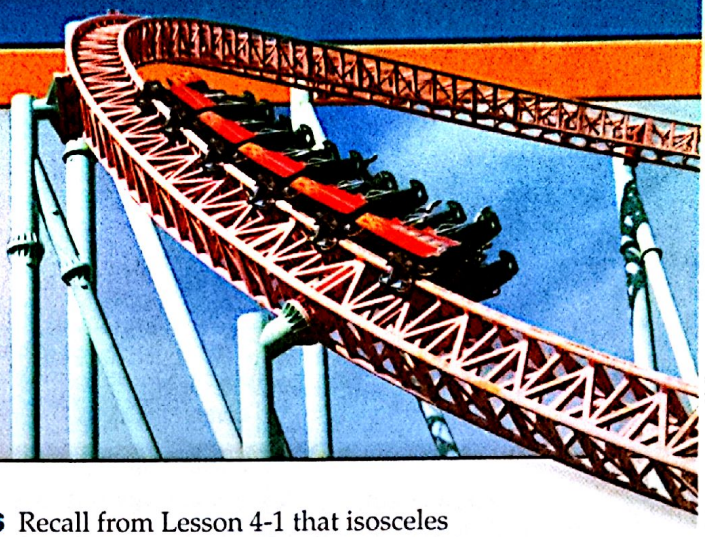
LESSON 4-6

Isosceles and Equilateral Triangles

Then
- You identified isosceles and equilateral triangles. (Lesson 4-1)

Now
- 1 Use properties of isosceles triangles.
- 2 Use properties of equilateral triangles.

Why?
- The tracks on the roller coaster have triangular reinforcements between the tracks for support and stability. The triangle supports in the photo are isosceles triangles.



New Vocabulary
- legs of an isosceles triangle
- vertex angle
- base angles

Tennessee Curriculum Standards

✔ **3108.4.10** Identify and apply properties and relationships of special figures.

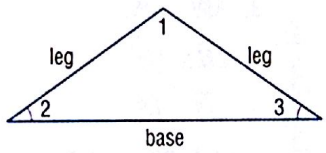
SPI 3108.4.3 Identify, describe and/or apply the relationships and theorems involving different types of triangles, quadrilaterals and other polygons.

1 Properties of Isosceles Triangles

Recall from Lesson 4-1 that isosceles triangles have at least two congruent sides. The parts of an isosceles triangle have special names.

The two congruent sides are called the **legs of an isosceles triangle**, and the angle with sides that are the legs is called the **vertex angle**. The side of the triangle opposite the vertex angle is called the *base*. The two angles formed by the base and the congruent sides are called the **base angles**.

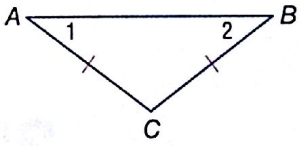
$\angle 1$ is the vertex angle.
$\angle 2$ and $\angle 3$ are the base angles.



Theorems Isosceles Triangle

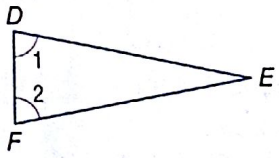
4.10 Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.



4.11 Converse of Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\overline{FE} \cong \overline{DE}$.



You will prove Theorem 4.11 in Exercise 37.

PT

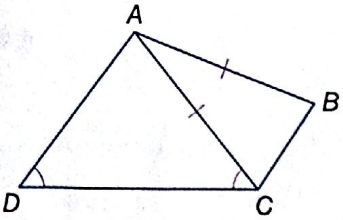
Example 1 Congruent Segments and Angles

a. Name two unmarked congruent angles.

$\angle ACB$ is opposite $\overline{AB}$ and $\angle B$ is opposite $\overline{AC}$, so $\angle ACB \cong \angle B$.

b. Name two unmarked congruent segments.

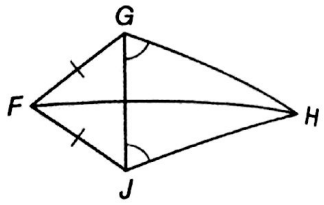
$\overline{AD}$ is opposite $\angle ACD$ and $\overline{AC}$ is opposite $\angle D$, so $\overline{AD} \cong \overline{AC}$.



GuidedPractice

1A. Name two unmarked congruent angles.

1B. Name two unmarked congruent segments.

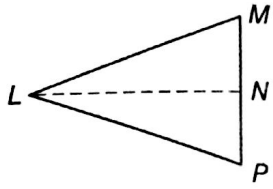


To prove the Isosceles Triangle Theorem, draw an auxiliary line and use the two triangles formed.

Proof Isosceles Triangle Theorem

Given: $\triangle LMP$; $\overline{LM} \cong \overline{LP}$

Prove: $\angle M \cong \angle P$



Proof:

Statements	Reasons
1. Let N be the midpoint of $\overline{MP}$.	1. Every segment has exactly one midpoint.
2. Draw an auxiliary segment $\overline{LN}$.	2. Two points determine a line.
3. $\overline{MN} \cong \overline{PN}$	3. Midpoint Theorem
4. $\overline{LN} \cong \overline{LN}$	4. Reflexive Property of Congruence
5. $\overline{LM} \cong \overline{LP}$	5. Given
6. $\triangle LMN \cong \triangle LPN$	6. SSS
7. $\angle M \cong \angle P$	7. CPCTC

2 Properties of Equilateral Triangles The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

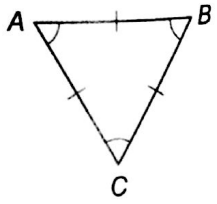
ReviewVocabulary

Equilateral Triangle a triangle with three congruent sides (Lesson 4-1)

Corollaries Equilateral Triangle

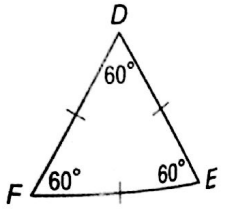
4.3 A triangle is equilateral if and only if it is equiangular.

Example If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.



4.4 Each angle of an equilateral triangle measures 60.

Example If $\overline{DE} \cong \overline{EF} \cong \overline{FE}$, then $m\angle A = m\angle B = m\angle C = 60$.

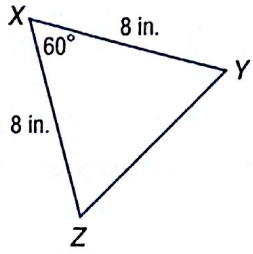


You will prove Corollaries 4.3 and 4.4 in Exercises 35 and 36.

Example 2 Find Missing Measures

Find each measure.

a. $m\angle Y$



Since $XY = XZ$, $\overline{XY} \cong \overline{XZ}$. By the Isosceles Triangle Theorem, base angles Z and Y are congruent, so $m\angle Z = m\angle Y$. Use the Triangle Sum Theorem to write and solve an equation to find $m\angle Y$.

$m\angle X + m\angle Y + m\angle Z = 180$	Triangle Sum Theorem
$60 + m\angle Y + m\angle Y = 180$	$m\angle X = 60$, $m\angle Z = m\angle Y$
$60 + 2(m\angle Y) = 180$	Simplify.
$2(m\angle Y) = 120$	Subtract 60 from each side.
$m\angle Y = 60$	Divide each side by 2.

b. YZ

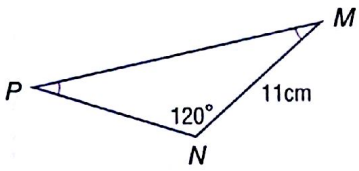
$m\angle Z = m\angle Y$, so $m\angle Z = 60$ by substitution. Since $m\angle X = 60$, all three angles measure 60, so the triangle is equiangular. Because an equiangular triangle is also equilateral, $XY = XZ = ZY$. Since $XY = 8$ inches, $YZ = 8$ inches by substitution.

StudyTip
Isosceles Triangles As you discovered in Example 2, any isosceles triangle that has one 60° angle must be an equilateral triangle.

GuidedPractice

2A. $m\angle M$

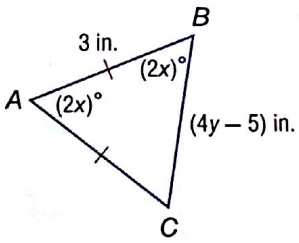
2B. PN



You can use the properties of equilateral triangles and algebra to find missing values.

Example 3 Find Missing Values

ALGEBRA Find the value of each variable.



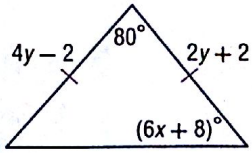
Since $\angle B = \angle A$, $\overline{AC} \cong \overline{BC}$ by the Converse of the Isosceles Triangle Theorem. All of the sides of the triangle are congruent, so the triangle is equilateral. Each angle of an equilateral triangle measures 60°, so $2x = 60$ and $x = 30$.

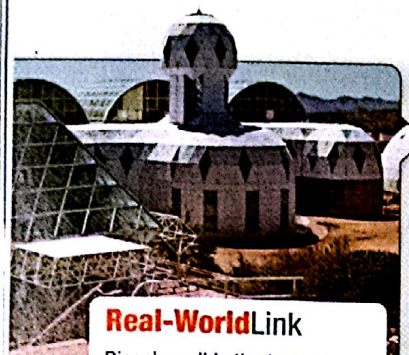
The triangle is equilateral, so all of the sides are congruent, and the lengths of all of the sides are equal.

$AB = BC$	Definition of equilateral triangle
$3 = 4y - 5$	Substitution
$8 = 4y$	Add 5 to each side.
$2 = y$	Divide each side by 4.

GuidedPractice

3. Find the value of each variable.





Real-World Link

Biosphere II is the largest totally enclosed ecosystem ever built, covering 3.14 acres in Oracle, Arizona. The controlled-environment facility is 91 feet at its highest point, and it has 6500 windows that enclose a volume of 7.2 million cubic feet.

Source: University of Arizona

Real-World Example 4 Apply Triangle Congruence

ENVIRONMENT Refer to the photo of Biosphere II at the right. $\triangle ACE$ is an equilateral triangle. F is the midpoint of $\overline{AE}$, D is the midpoint of $\overline{EC}$, and B is the midpoint of $\overline{CA}$. Prove that $\triangle FBD$ is also equilateral.

Given: $\triangle ACE$ is equilateral. F is the midpoint of $\overline{AE}$, D is the midpoint of $\overline{EC}$, and B is the midpoint of $\overline{CA}$.

Prove: $\triangle FBD$ is equilateral.

Proof:

Statements	Reasons
1. $\triangle ACE$ is equilateral.	1. Given
2. F is the midpoint of AE, D is the midpoint of EC, and B is the midpoint of CA.	2. Given
3. $m\angle A = 60, m\angle C = 60, m\angle E = 60$	3. Each angle of an equilateral triangle measures 60.
4. $\angle A \cong \angle C \cong \angle E$	4. Definition of congruence and substitution
5. $\overline{AE} \cong \overline{EC} \cong \overline{CA}$	5. Definition of equilateral triangle
6. $AE = EC = CA$	6. Definition of congruence
7. $\overline{AF} \cong \overline{FE}, \overline{ED} \cong \overline{DC}, \overline{CB} \cong \overline{BA}$	7. Midpoint Theorem
8. $AF = FE, ED = DC, CB = BA$	8. Definition of congruence
9. $AF + FE = AE, ED + DC = EC, CB + BA = CA$	9. Segment Addition Postulate
10. $AF + AF = AE, FE + FE = AE, ED + ED = EC, DC + DC = EC, CB + CB = CA, BA + BA = CA$	10. Substitution
11. $2AF = AE, 2FE = AE, 2ED = EC, 2DC = EC, 2CB = CA, 2BA = CA$	11. Addition Property
12. $2AF = AE, 2FE = AE, 2ED = AE, 2DC = AE, 2CB = AE, 2BA = AE$	12. Substitution Property
13. $2AF = 2ED = 2CB, 2FE = 2DC = 2BA$	13. Transitive Property
14. $AF = ED = CB, FE = DC = BA$	14. Division Property
15. $\overline{AF} \cong \overline{ED} \cong \overline{CB}, \overline{FE} \cong \overline{DC} \cong \overline{BA}$	15. Definition of congruence
16. $\triangle AFB \cong \triangle EDF \cong \triangle CBD$	16. SAS
17. $\overline{DF} \cong \overline{FB} \cong \overline{BD}$	17. CPCTC
18. $\triangle FBD$ is equilateral.	18. Definition of equilateral triangle

GuidedPractice

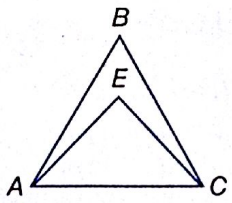
4. Given that $\triangle ACE$ is equilateral, $\overline{FB} \parallel \overline{EC}$, $\overline{FD} \parallel \overline{BC}$, $\overline{BD} \parallel \overline{EF}$, and D is the midpoint of $\overline{EC}$, prove that $\triangle FED \cong \triangle BDC$.

Check Your Understanding

= Step-by-Step Solutions begin on page R20.

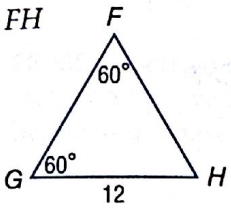
Example 1 Refer to the figure at the right.

1. If $\overline{AB} \cong \overline{CB}$, name two congruent angles.
2. If $\angle EAC \cong \angle ECA$, name two congruent segments.

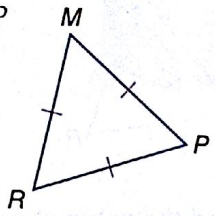


Example 2 Find each measure.

3. FH

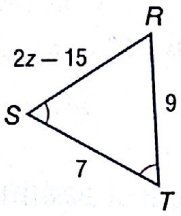


4. $m\angle MRP$

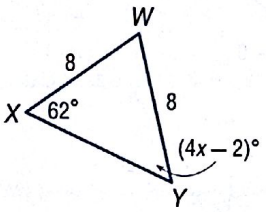


Example 3 **ALGEBRA** Find the value of each variable.

5.



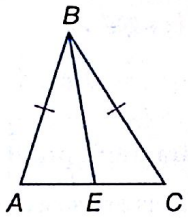
6.



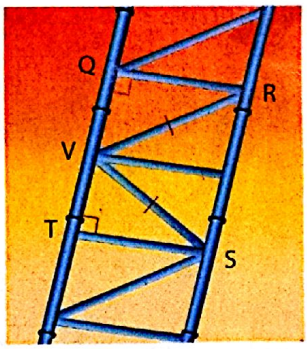
Example 4 7. **PROOF** Write a two-column proof.

Given: $\triangle ABC$ is isosceles; $\overline{EB}$ bisects $\angle ABC$.

Prove: $\triangle ABE \cong \triangle CBE$



8. **ROLLER COASTERS** The roller coaster track shown in the photo on page 283 appears to be composed of congruent triangles. A portion of the track is shown.



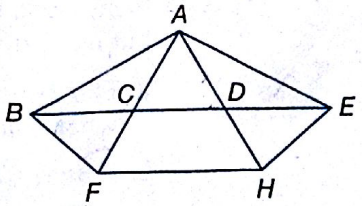
   a. If $\overline{QR}$ and $\overline{ST}$ are perpendicular to $\overline{QT}$, $\triangle VSR$ is isosceles with base $\overline{SR}$, and $\overline{QT} \parallel \overline{SR}$, prove that $\triangle RQV \cong \triangle STV$.

   b. If $VR = 2.5$ meters and $QR = 2$ meters, find the distance between $\overline{QR}$ and $\overline{ST}$. Explain your reasoning.

Practice and Problem Solving

Extra Practice begins on page 969.

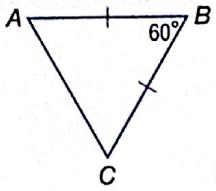
Example 1 Refer to the figure at the right.



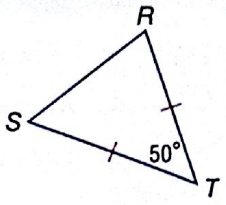
9. If $\overline{AB} \cong \overline{AE}$, name two congruent angles.
10. If $\angle ABF \cong \angle AFB$, name two congruent segments.
11. If $\overline{CA} \cong \overline{DA}$, name two congruent angles.
12. If $\angle DAE \cong \angle DEA$, name two congruent segments.
13. If $\angle BCF \cong \angle BFC$, name two congruent segments.
14. If $\overline{FA} \cong \overline{AH}$, name two congruent angles.

Example 2 **Find each measure.**

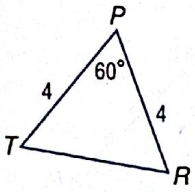
15. $m\angle BAC$



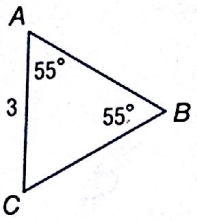
16. $m\angle SRT$



17. TR

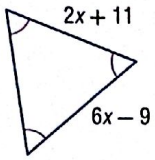


18. CB



Example 3 **ALGEBRA Find the value of each variable.**

19.



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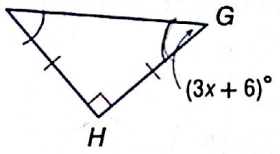

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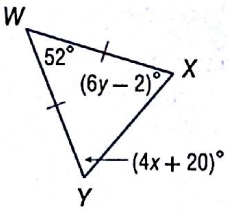

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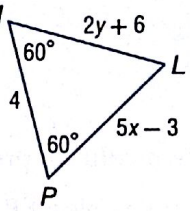
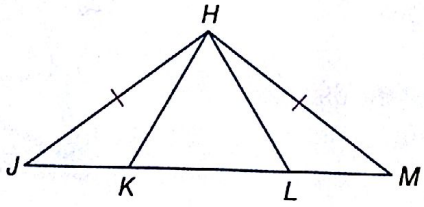
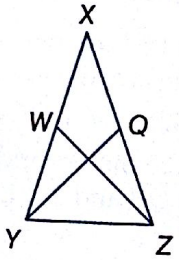


Example 4 **PROOF Write a paragraph proof.**

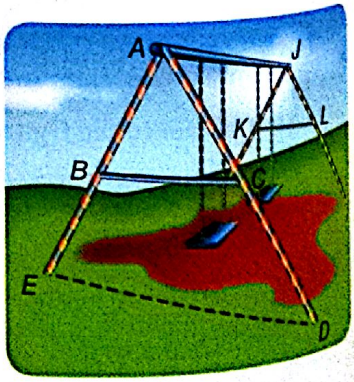
23. **Given:** $\triangle HJM$ is an isosceles triangle, and $\triangle HKL$ is an equilateral triangle. $\angle JKH$ and $\angle HKL$ and $\angle HLK$ and $\angle MLH$ are supplementary.
Prove: $\angle JHK \cong \angle MHL$



24. **Given:** $\overline{XY} \cong \overline{XZ}$
W is the midpoint of $\overline{XY}$.
Q is the midpoint of $\overline{XZ}$.
Prove: $\overline{WZ} \cong \overline{QY}$



25. **BABYSITTING** While babysitting her neighbor's children, Elisa observes that the supports on either side of a park swing set form two sets of triangles. Using a jump rope to measure, Elisa is able to determine that $\overline{AB} \cong \overline{AC}$, but $\overline{BC} \ncong \overline{AB}$.



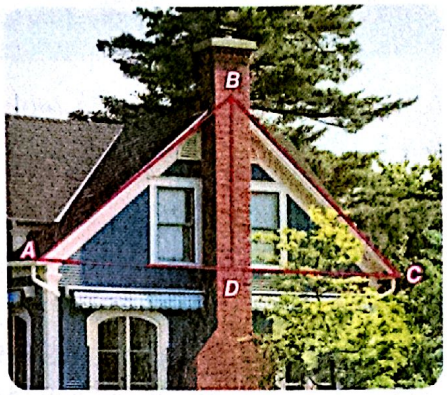
a. Elisa estimates $m\angle BAC$ to be 50. Based on this estimate, what is $m\angle ABC$? Explain.

b. If $\overline{BE} \cong \overline{CD}$, show that $\triangle AED$ is isosceles.

c. If $\overline{BC} \parallel \overline{ED}$ and $\overline{ED} \cong \overline{AD}$, show that $\triangle AED$ is equilateral.

d. If $\triangle JKL$ is isosceles, what is the minimum information needed to prove that $\triangle ABC \cong \triangle JLK$? Explain your reasoning.

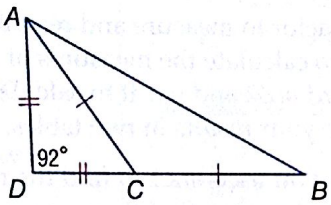
26. **CHIMNEYS** In the picture, $\overline{BD} \perp \overline{AC}$ and $\triangle ABC$ is an isosceles triangle with base $\overline{AC}$. Show that the chimney of the house, represented by $\overline{BD}$, bisects the angle formed by the sloped sides of the roof, $\angle ABC$.



27. **CONSTRUCTION** Construct three different isosceles right triangles. Explain your method. Then verify your constructions using measurement and mathematics.

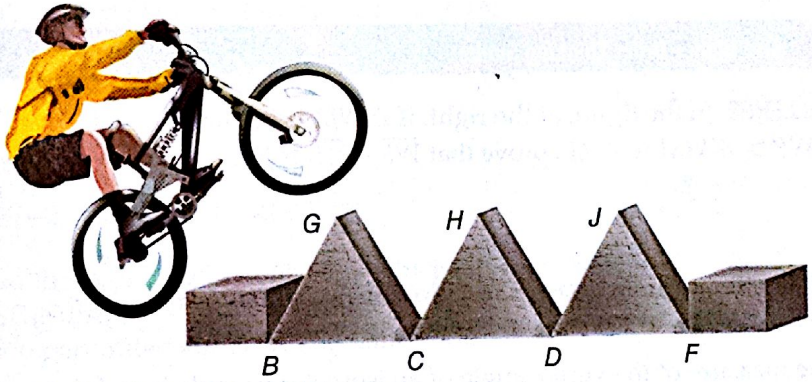
28. **PROOF** Based on your construction in Exercise 27, make and prove a conjecture about the relationship between the base angles of an isosceles right triangle.

Find each measure.

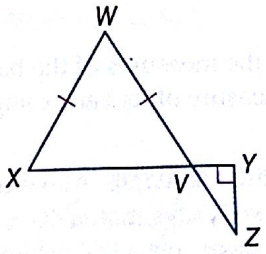


29. $m\angle CAD$
30. $m\angle ACD$
31. $m\angle ACB$
32. $m\angle ABC$

33. **FITNESS** In the diagram, the rider will use his bike to hop across the tops of each of the concrete solids shown. If each triangle is isosceles with vertex angles G, H, and J, and $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$, show that the distance from B to F is three times the distance from D to F.



34. **Given:** $\triangle XWV$ is isosceles; $\overline{ZY} \perp \overline{YV}$.
Prove: $\angle X$ and $\angle YZV$ are complementary.

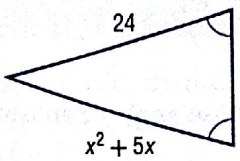


PROOF Write a two-column proof of each corollary or theorem.

35. Corollary 4.3
36. Corollary 4.4
37. Theorem 4.11

Find the value of each variable.

38.



39.