LESSON 12-4

Volumes of Prisms and Cylinders

Then

- You found surface areas of prisms and cylinders. (Lesson 12-2)

Now

- 1 Find volumes of prisms.
- 2 Find volumes of cylinders.

Why?

- Planters come in a variety of shapes and sizes. You can approximate the amount of soil needed to fill a planter by finding the volume of the three-dimensional figure that it most resembles.

Tennessee Curriculum Standards

CLE 3108.4.6 Generate formulas for perimeter, area, and volume, including their use, dimensional analysis, and applications.

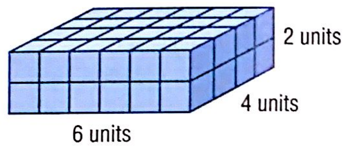
✔ **3108.4.25** Use properties of prisms, pyramids, cylinders, cones, spheres, and hemispheres to solve problems.

SPI 3108.4.9 Use right triangle trigonometry and cross-sections to solve problems involving surface areas and/or volumes of solids. *Also addresses SPI 3108.4.14.*

1 Volume of Prisms

Recall that the volume of a solid is the measure of the amount of space the solid encloses. Volume is measured in cubic units.

The rectangular prism at the right has $6 \cdot 4$ or 24 cubic units in the bottom layer. Since there are two layers, the total volume is $24 \cdot 2$ or 48 cubic units.

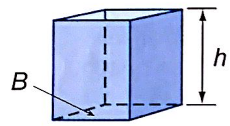


KeyConcept Volume of a Prism

Words The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height of the prism.

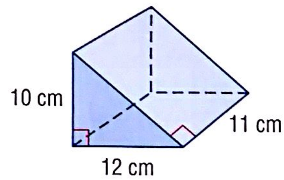
Symbols $V = Bh$

Model



Example 1 Volume of a Prism

Find the volume of the prism.



Step 1 Find the area of the base B.

$B = \frac{1}{2}bh$ — Area of a triangle

$= \frac{1}{2}(12)(10)$ or 60 — $b = 12$ and $h = 10$

Step 2 Find the volume of the prism.

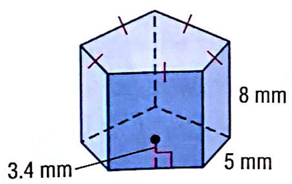
$V = Bh$ — Volume of a prism

$= 60(11)$ or 660 — $B = 60$ and $h = 11$

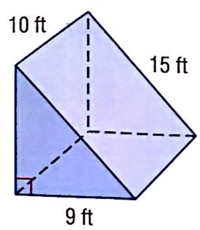
The volume of the prism is 660 cubic centimeters.

GuidedPractice

1A.

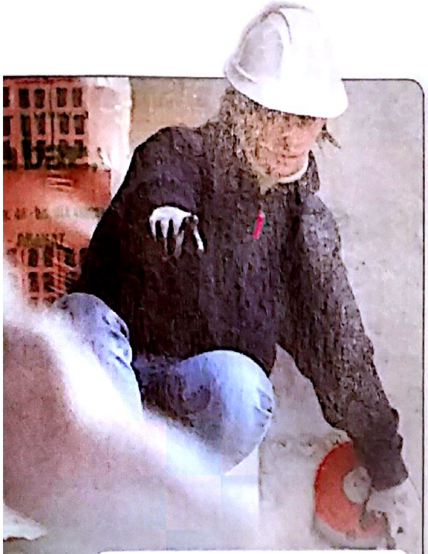


1B.



2 Volume of Cylinders

Like a prism, the volume of a cylinder is the product of the area of the base and the height.



Real-World Career

Architectural Engineer
An architectural engineer applies the technical skills of engineering to the design, construction, operation, maintenance, and renovation of buildings.

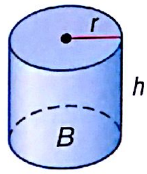
Architectural engineers are required to have a bachelor's degree in engineering along with specialized coursework. Refer to Exercise 35.

KeyConcept Volume of a Cylinder

Words The volume V of a cylinder is $V = Bh$ or $V = \pi r^2 h$, where B is the area of the base, h is the height of the cylinder, and r is the radius of the base.

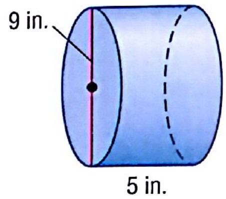
Symbols $V = Bh$ or $V = \pi r^2 h$

Model



Example 2 Volume of a Cylinder

Find the volume of the cylinder at the right.



Estimate: $V \approx 3 \cdot 5^2 \cdot 5$ or 375 in^3

$V = \pi r^2 h$ Volume of a cylinder

$= \pi(4.5)^2(5)$ $r = 4.5$ and $h = 5$

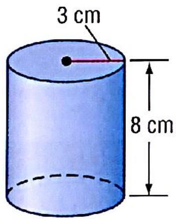
≈ 318.1 Use a calculator.

The volume of the cylinder is about 318.1 cubic inches. This is fairly close to the estimate, so the answer is reasonable.

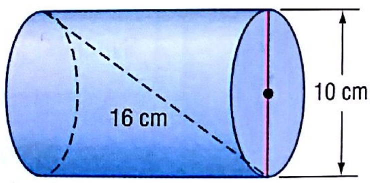
GuidedPractice

Find the volume of each cylinder. Round to the nearest tenth.

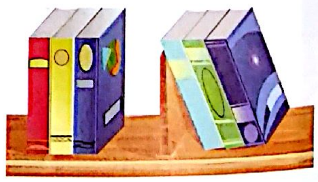
2A.



2B.



The first group of books at the right represents a right prism. The second group represents an oblique prism. Both groups have the same number of books. If all the books are the same size, then the volume of both groups is the same.



This demonstrates the following principle, which applies to all solids.

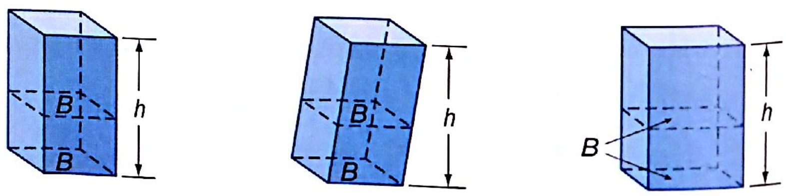
WatchOut!

Cross-Sectional Area
For solids with the same height to have the same volume, their cross-sections must have the same area. The cross sections of the different solids do not have to be congruent polygons.

KeyConcept Cavalieri's Principle

Words If two solids have the same height h and the same cross-sectional area B at every level, then they have the same volume.

Models



These prisms all have a volume of Bh.

Problem-Solving Tip

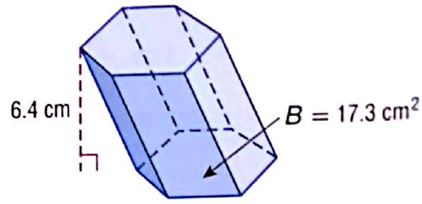
Make a Model When solving problems involving volume of solids, one way to help you visualize the problem is to make a model of the solid.

Example 3 Volume of an Oblique Solid

Find the volume of an oblique hexagonal prism if the height is 6.4 centimeters and the base area is 17.3 square centimeters.

$V = Bh$	Volume of a prism
$= 17.3(6.4)$	$B = 17.3$ and $h = 6.4$
$= 110.72$	Simplify.

The volume is 110.72 cubic centimeters.



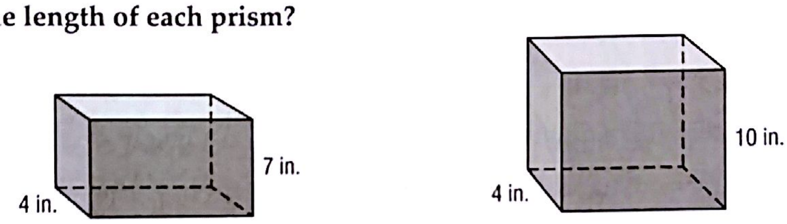
Guided Practice

3. Find the volume of an oblique cylinder that has a radius of 5 feet and a height of 3 feet. Round to the nearest tenth.

SPI 3108.4.9

Test Example 4

Prisms A and B have the same length and width, but different heights. If the volume of Prism B is 150 cubic inches greater than the volume of Prism A, what is the length of each prism?



Prism A　　　Prism B

A 10 in.　　**B** $11\frac{1}{2}$ in.　　**C** 12 in.　　**D** $12\frac{1}{2}$ in.

Read the Test Item

You know two dimensions of each solid and that the difference between their volumes is 150 cubic inches.

Solve the Test Item

Volume of Prism B − Volume of Prism A = 150	Write an equation.
$4\ell \cdot 10 - 4\ell \cdot 7 = 150$	Use $V = Bh$.
$12\ell = 150$	Simplify.
$\ell = 12\frac{1}{2}$	Divide each side by 12.

The length of each prism is $12\frac{1}{2}$ inches. The correct answer is D.

Test-Taking Tip

Writing Equations By choosing a variable to represent the unknown and setting up an equation, you make it easier to check for mistakes as you work out a problem.

Guided Practice

4. The containers at the right are filled with popcorn. About how many times as much popcorn does the larger container hold?

F 1.6 times as much

G 2.5 times as much

H 3.3 times as much

J 5.0 times as much