2-1 Inductive Reasoning and Conjecture

Then
- You used data to find patterns and make predictions.

Now
1. Make conjectures based on inductive reasoning.
2. Find counterexamples.

Why?
- Market research is conducted by an analyst to answer specific questions about products. For example, a company that creates video games might hire focus group testers to play an unreleased video game. The process of using patterns to analyze the effectiveness of a product involves inductive reasoning.

New Vocabulary
- inductive reasoning
- conjecture
- counterexample

Tennessee Curriculum Standards

CLE 3108.1.3 Develop inductive and deductive reasoning to independently make and evaluate mathematical arguments and construct appropriate proofs; include various types of reasoning, logic, and intuition.

✔ **3108.1.6** Use inductive reasoning to write conjectures and/or conditional statements.

✔ **3108.4.18** Use counterexamples, when appropriate, to disprove a statement. *Also addresses CLE 3108.4.3.*

1 Make Conjectures

Inductive reasoning is reasoning that uses a number of specific examples to arrive at a conclusion. When you assume that an observed pattern will continue, you are applying inductive reasoning. A concluding statement reached using inductive reasoning is called a **conjecture**.

Example 1 Patterns and Conjecture

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

a. Movie show times: 8:30 A.M., 9:45 A.M., 11:00 A.M., 12:15 P.M., . . .

Step 1 Look for a pattern.

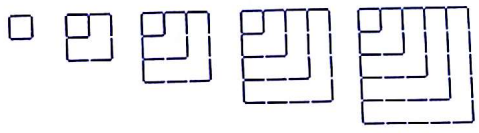
8:30 A.M., 9:45 A.M., 11:00 A.M., 12:15 P.M., . . .

+1 hr 15 min +1 hr 15 min +1 hr 15 min

Step 2 Make a conjecture.

The show time is 1 hour and fifteen minutes greater than the previous show time. The next show time will be 12:15 P.M. + 1:15 or 1:30 P.M.

b.



4 10 18 28 40 . .

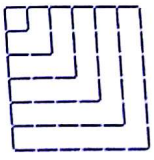
Step 1 4, 10, 18, 28, 40

+6 +8 +10 +12

The numbers increase by 6, 8, 10, and 12.

Step 2 The next figure will increase by 12 + 2 or 14 segments. So, the next figure will have 40 + 14 or 54 segments.

CHECK Draw the next figure to check your conjecture ✔



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ReviewVocabulary

sequence an arrangement of numbers or items in a particular order

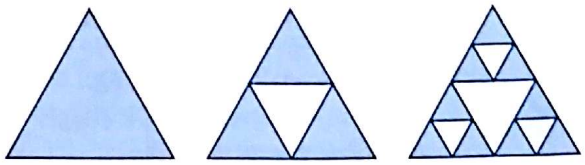
GuidedPractice

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1A. Follow-up visits: Dec., May, Oct., Mar., . . .

1B. 10, 4, −2, −8, . . .

1C.



To make some algebraic and geometric conjectures, you will need to provide examples.

StudyTip

Supporting Examples and Proofs Examples that support a conjecture are not enough to show that a conjecture is true. To show that an algebraic or geometric conjecture is true, you must offer a logical argument called a proof. You will learn more about proofs in Lesson 2-5.

Example 2 Algebraic and Geometric Conjectures

Make a conjecture about each value or geometric relationship. List or draw some examples that support your conjecture.

a. the sum of two odd numbers

Step 1 List examples.

$1 + 3 = 4$ $1 + 5 = 6$ $3 + 5 = 8$ $7 + 9 = 16$

Step 2 Look for a pattern.

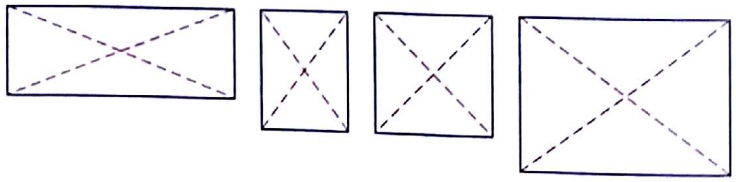
Notice that the sums 4, 6, 8, and 16 are all even numbers.

Step 3 Make a conjecture.

The sum of two odd numbers is an even number.

b. segments joining opposite vertices of a rectangle

Step 1



Step 2 Notice that the segments joining opposite vertices of each rectangle appear to have the same measure. Use a ruler or compass to confirm this.

Step 3 Conjecture: the segments joining opposite vertices of a rectangle are congruent.

GuidedPractice

2A. the sum of two even numbers

2B. the relationship between AB and EF, if $AB = CD$ and $CD = EF$

2C. the sum of the squares of two consecutive natural numbers

Real-world conjectures are often made based on data gathered about a specific topic of interest.

Real-World Career

Hair Stylist Hair stylists work in salons where various services, including skin and nail treatments, may be provided in addition to hair care. About 48% of hair stylists are self-employed and own their own businesses. Hair stylists must attend cosmetology school and obtain a license.

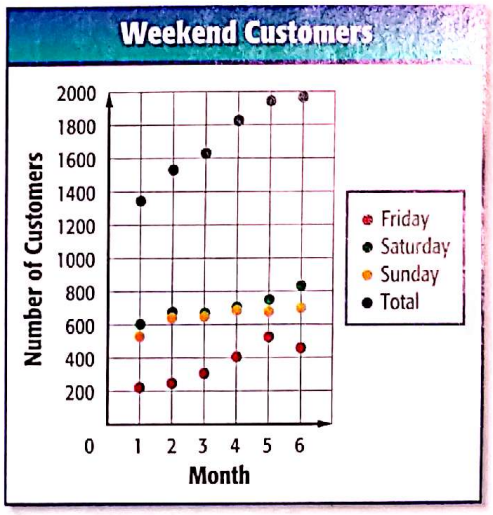
Real-World Example 3 Make Conjectures from Data

BUSINESS **The owner of a hair salon collected data on the number of customers her salon had each Friday, Saturday, and Sunday for 6 months to decide whether she should increase the number of stylists working each weekend. The data she collected are shown below.**

Number of Customers on the Weekend						
Day	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Friday	225	255	321	406	540	450
Saturday	603	658	652	712	746	832
Sunday	552	635	642	692	685	705
Total	1380	1548	1615	1810	1971	1987

a. Make a statistical graph that best displays the data.

Since you want to look for a pattern over time, use a scatter plot to display the data. Label the horizontal axis with the months and the vertical axis with the number of customers. Plot each set of data using a different color and include a legend.



b. Make a conjecture based on the data and explain how this conjecture is supported by your graph.

Look for patterns in the data. The number of customers on each day usually increases each month, and the total number of customers increases every single month.

Survey data supports a conjecture that the amount of business on the weekends has increased, so the owner should schedule more stylists to work on those days.

GuidedPractice

3. POSTAGE The table at the right shows the price of postage for the years 1982 through 2009.

Year	Rate (cents)
1982	20
1987	22
1992	29
1997	32
2002	37
2007	41
2009	44

A. Construct a statistical graph that best displays the data.

B. Predict the postage rate in 2015 based on the graph.

C. Does it make sense that the pattern of the data will continue over time? If not, how will it change? Explain your reasoning.

2 Find Counterexamples To show that a conjecture is true for all cases, you must prove it. It takes only one false example, however, to show that a conjecture is not true. This false example is called a **counterexample**, and it can be a number, a drawing, or a statement.

VocabularyLink
Counterexample
Everyday Use The prefix *counter-* means *the opposite of.*
Math Use A counterexample is the opposite of an example.

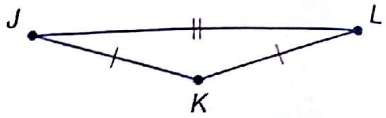
Example 4 Find Counterexamples

Find a counterexample to show that each conjecture is false.

a. If n is a real number, then $n^2 > n$.

When n is 1, the conjecture is false, since $1^2 \not> 1$.

b. If $JK = KL$, then K is the midpoint of $\overline{JL}$.



When J, K, and L are noncollinear, the conjecture is false. In the figure, $JK = KL$, but K is not the midpoint of $\overline{JL}$.

GuidedPractice

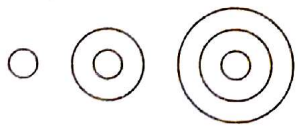
4A. If n is a real number, then $-n$ is a negative.

4B. If $\angle ABC \cong \angle DBE$, then $\angle ABC$ and $\angle DBE$ are vertical angles.

Check Your Understanding

= Step-by-Step Solutions begin on page R20

Example 1 **Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.**

1. Costs: \$4.50, \$6.75, \$9.00 . . .
2. Appointment times: 10:15 A.M., 11:00 A.M., 11:45 A.M., . . .
3. 
4. 
5. 3, 3, 6, 9, 15, . . .
6. 2, 6, 14, 30, 62, . . .

Example 2 **Make a conjecture about each value or geometric relationship.**

7. the product of two even numbers
8. the relationship between a and b if $a + b = 0$
9. the relationship between the set of points in a plane equidistant from point A
10. the relationship between $\overline{AP}$ and $\overline{PB}$ if M is the midpoint of $\overline{AB}$ and P is the midpoint of $\overline{AM}$

Example 3

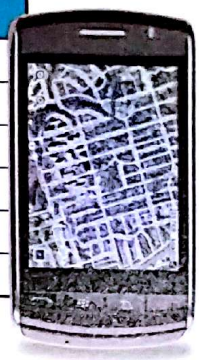
11. CELL PHONES Refer to the table of the number of wireless subscriptions in the United States by year.

a. Make a graph that shows U.S. wireless use from 2002 to 2007.

b. Make a conjecture about U.S. wireless use in 2012.

U.S. Wireless Subscribership	
Year	Subscribers (Millions)
2002	140.8
2003	158.7
2004	182.1
2005	207.9
2006	233.0
2007	255.4

Source: Cellular Telecommunications and Internet Association



Example 4

Find a counterexample to show that each conjecture is false.

12. If $\angle A$ and $\angle B$ are complementary angles, then they share a common side.

13 If a ray intersects a segment at its midpoint, then the ray is perpendicular to the segment.

Practice and Problem Solving

Extra Practice begins on page 969.

Example 1

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

14. 0, 2, 4, 6, 8

15. 3, 6, 9, 12, 15

16. 4, 8, 12, 16, 20

17. 2, 22, 222, 2222

18. 1, 4, 9, 16

19. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

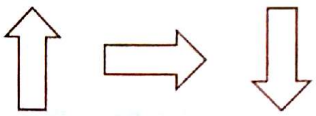
20. Arrival times: 3:00 P.M., 12:30 P.M., 10:00 A.M., . . .

21. Percent humidity: 100%, 93%, 86%, . . .

22. Work-out days: Sunday, Tuesday, Thursday, . . .

23. Club meetings: January, March, May, . . .

24.



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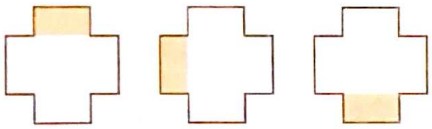

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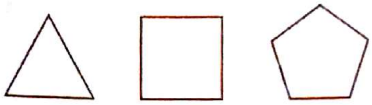

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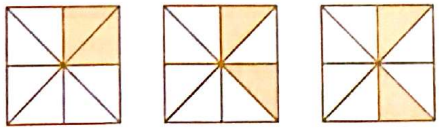


28. FITNESS Gabriel started training with the track team five weeks ago. During the first week, he ran 0.5 mile at each practice. The next three weeks he ran 0.75 mile, 1 mile, and 1.25 miles at each practice. If he continues this pattern, how many miles will he be running at each practice during the 7th week?

29. CONSERVATION When there is a shortage of water, some municipalities limit the amount of water each household is allowed to consume. Most cities that experience water restrictions are in the western and southern parts of the United States. Make a conjecture about why water restrictions occur in these areas.

30. VOLUNTEERING Carrie collected canned food for a homeless shelter in her area each day for one week. On day one, she collected 7 cans of food. On day two, she collected 8 cans. On day three she collected 10 cans. On day four, she collected 13 cans. If Carrie wanted to give at least 100 cans of food to the shelter and this pattern of can collecting continued, did she meet her goal?

Example 2 **Make a conjecture about each value or geometric relationship.**

31. the product of two odd numbers
32. the product of two and a number, plus one
33. the relationship between a and c if $ab = bc$, $b \neq 0$
34. the relationship between a and b if $ab = 1$
35. the relationship between $\overline{AB}$ and the set of points equidistant from A and B
36. the relationship between the angles of a triangle with all sides congruent
37. the relationship between the areas of a square with side x and a rectangle with sides x and $2x$
38. the relationship between the volume of a prism and a pyramid with the same base

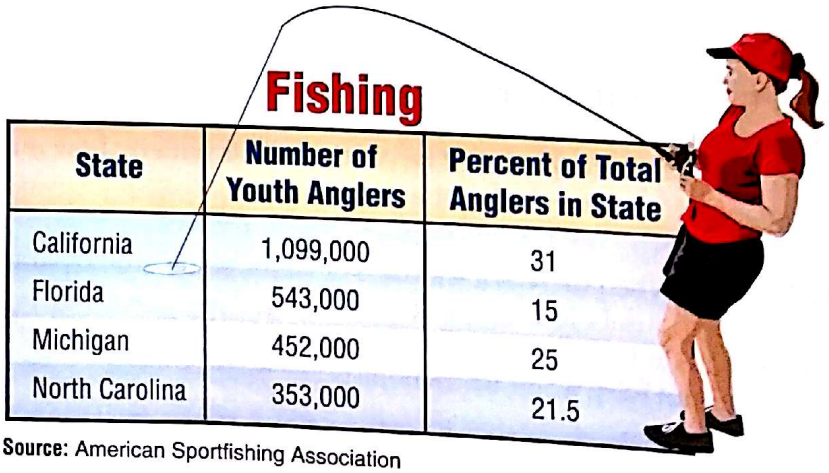
Example 3

39. **SPORTS** Refer to the table of Americans over the age of 7 that played hockey.
    a. Construct a statistical graph that best displays the data.
    b. Make a conjecture based on the data and explain how this conjecture is supported by your graph.

Year	Number of Participants (millions)
2000	1.9
2002	2.1
2004	2.4
2006	2.6

Example 4 **Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.**

40. If n is a prime number, then $n + 1$ is not prime.
41. If x is an integer, then $-x$ is positive.
42. If $\angle 2$ and $\angle 3$ are supplementary angles, then $\angle 2$ and $\angle 3$ form a linear pair.
43. If you have three points A, B, and C, then A, B, C are noncollinear.
44. If in $\triangle ABC$, $(AB)^2 + (BC)^2 = (AC)^2$, then $\triangle ABC$ is a right triangle.
45. If the area of a rectangle is 20 square meters, then the length is 10 meters and the width is 2 meters.
46. **NATURE** Refer to the table and find a counterexample for each statement.



Fishing

State	Number of Youth Anglers	Percent of Total Anglers in State
California	1,099,000	31
Florida	543,000	15
Michigan	452,000	25
North Carolina	353,000	21.5

Source: American Sportfishing Association

a. The number of youth anglers in a state is less than one fourth of the total anglers in that state.
b. Each state listed has at least 3,000,000 total anglers.

47. **GOLDBACH'S CONJECTURE** Goldbach's conjecture states that every even number greater than 2 can be written as the sum of two primes. For example, $4 = 2 + 2$, $6 = 3 + 3$, and $8 = 3 + 5$.

a. Show that the conjecture is true for the even numbers from 10 to 20.

b. Given the conjecture *All odd numbers greater than 2 can be written as the sum of two primes*, is the conjecture *true* or *false*? Give a counterexample if the conjecture is false.

48. **SEGMENTS** Two collinear points form one segment, as shown for $\overline{AB}$. If a collinear point is added to $\overline{AB}$, the three collinear points form three segments.

a. How many distinct segments are formed by four collinear points? by five collinear points?

b. Make a conjecture about the number of distinct segments formed by n collinear points.

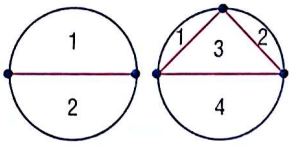
c. Test your conjecture by finding the number of distinct segments formed by six points.

49. **MEASUREMENT** Using dynamic geometry software, Nora calculates the perimeter P and area A of a regular hexagon with a side length of 2 units. The change to the perimeter and area after three doublings of this side length are listed in the table. Analyze the patterns in the table. Then make a conjecture as to the effects on the perimeter and area of a regular hexagon when the side length is doubled. Explain.

Side (units)	P (units)	A (units^2)
2	12	$6\sqrt{3}$
4	24	$24\sqrt{3}$
8	48	$96\sqrt{3}$
16	96	$384\sqrt{3}$

H.O.T. Problems Use Higher-Order Thinking Skills

50. **CHALLENGE** If you draw points on a circle and connect every pair of points, the circle is divided into regions. For example, two points form two regions and three points form four regions.



a. Make a conjecture about the relationship between the number of points on a circle and the number of regions formed in the circle.

b. Does your conjecture hold true when there are six points? Support your answer with a diagram.

51. **ERROR ANALYSIS** Juan and Jack are discussing prime numbers. Juan states a conjecture that all prime numbers are odd. Jack disagrees with the conjecture and states not all prime numbers are odd. Is either of them correct? Explain.

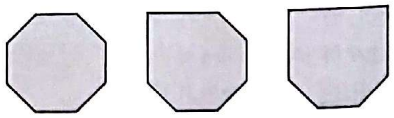
52. **OPEN ENDED** Write a number sequence that can be generated by two different patterns. Explain your patterns.

53. **REASONING** Consider the conjecture *If two points are equidistant from a third point, then the three points are collinear*. Is the conjecture *true* or *false*? If false, give a counterexample.

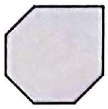
54. **WRITING IN MATH** Suppose you are conducting a survey. Choose a topic and write three questions you would include in your survey. How would you use inductive reasoning with your responses?

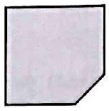
Standardized Test Practice

55. Look at the pattern below.



If the pattern continues, what will be the next shape?

A 

C 

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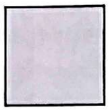
D 

56. GRIDDED RESPONSE What is the value of the expression below if $a = 10$ and $b = 1$?

$$2b + ab \div (a + b)$$

57. ALGEBRA A chemistry student mixed some 30% copper sulfate with some 40% copper sulfate solution to obtain 100 mL of a 32% copper sulfate solution. How much of the 30% copper sulfate solution did the student use in the mixture?

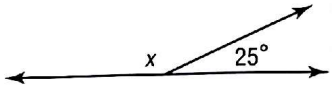
F 90 mL

G 80 mL

H 60 mL

J 20 mL

58. SAT/ACT Which of the following is equal to $2x$?



A 50°

B 78°

C 155°

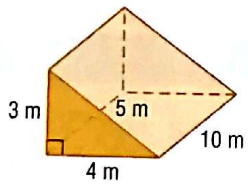
D 310°

E 360°

Spiral Review

Find the surface area and volume of each solid. (Lesson 1-7)

59.



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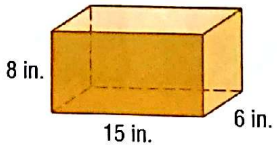

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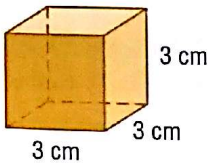


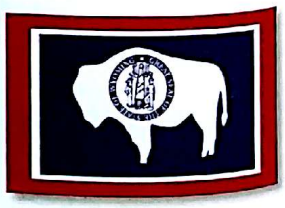
Find the perimeter of $\triangle ABC$ to the nearest hundredth, given the coordinates of its vertices. (Lesson 1-6)

62. $A(1, 6), B(1, 2), C(3, 2)$

63. $A(-3, 2), B(2, -9), C(0, -10)$

64. ALGEBRA The measures of two complementary angles are $16z - 9$ and $4z + 3$. Find the measures of the angles. (Lesson 1-5)

65. FLAGS The Wyoming state flag is shown at the right. Name the geometric term modeled by this flag: point, line, or plane. (Lesson 1-1)



66. ALGEBRA Evaluate $5|x + y| - 3|2 - z|$ if $x = 3$, $y = -4$, and $z = -5$. (Lesson 0-4)

Skills Review

ALGEBRA Determine which values in the replacement set make the inequality true. (Lesson 0-6)

67. $x - 3 > 12$
$\{6, 10, 14, 18\}$

68. $6 + x > 9$
$\{8, 6, 4, 2\}$

69. $2x - 4 > 10$
$\{5, 6, 7, 8\}$