LESSON 1-1

Points, Lines, and Planes

Then

- You used basic geometric concepts and properties to solve problems.

Now

1. Identify and model points, lines, and planes.
2. Identify intersecting lines and planes.

Why?

- On a subway map, the locations of stops are represented by *points*. The route the train can take is modeled by a series of connected paths that look like *lines*. The flat surface of the map on which these points and lines lie is representative of a *plane*.

New Vocabulary

- undefined term
- point
- line
- plane
- collinear
- coplanar
- intersection
- definition
- defined term
- space

Tennessee Curriculum Standards

SPI 3108.1.1 Give precise mathematical descriptions or definitions of geometric shapes in the plane and space.

CLE 3108.4.1 Develop the structures of geometry, such as lines, angles, planes, and planar figures, and explore their properties and relationships.

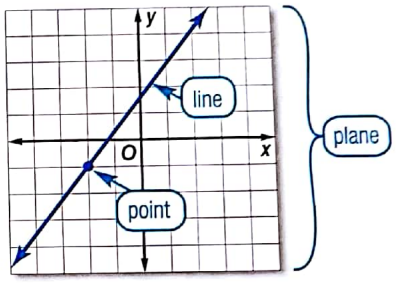
✓ 3108.4.6 Describe the intersection of lines (in the plane and in space), a line and a plane, or of two planes. *Also addresses ✓3108.1.2, ✓3108.1.4, and SPI 31008.4.2.*

1 Points, Lines, and Planes

Unlike the real-world objects that they model, shapes, points, lines, and planes do not have any actual size. In geometry, *point*, *line*, and *plane* are considered **undefined terms** because they are only explained using examples and descriptions.

You are already familiar with the terms point, line, and plane from algebra. You graphed on a coordinate *plane* and found ordered pairs that represented *points* on *lines*. In geometry, these terms have a similar meaning.

The phrase *exactly one* in a statement such as, "There is exactly one line through any two points," means that there is *one and only one*.



KeyConcept Undefined Terms

A **point** is a location. It has neither shape nor size.

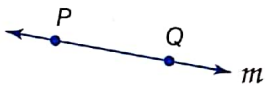
Named by a capital letter

Example point *A*

A **line** is made up of points and has no thickness or width. There is exactly one line through any two points.

Named by the letters representing two points on the line or a lowercase script letter

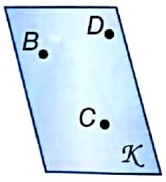
Example line *m*, line *PQ* or $\overleftrightarrow{PQ}$, line *QP* or $\overleftrightarrow{QP}$



A **plane** is a flat surface made up of points that extends infinitely in all directions. There is exactly one plane through any three points not on the same line.

Named by a capital script letter or by the letters naming three points that are not all on the same line

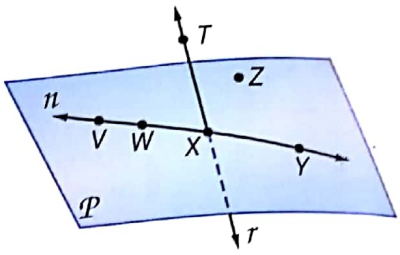
Example plane $\mathcal{K}$, plane *BCD*, plane *CDB*, plane *DCB*, plane *DBC*, plane *CBD*, plane *BDC*



Collinear points are points that lie on the same line. *Noncollinear* points do not lie on the same line. **Coplanar** points are points that lie in the same plane. *Noncoplanar* points do not lie in the same plane.

Example 1 Name Lines and Planes

Use the figure to name each of the following.



a. a line containing point W

The line can be named as line n, or any two of the four points on the line can be used to name the line.

$\overleftrightarrow{VW}$ $\overleftrightarrow{WV}$ $\overleftrightarrow{VX}$ $\overleftrightarrow{XV}$ $\overleftrightarrow{VY}$ $\overleftrightarrow{YV}$
$\overleftrightarrow{WX}$ $\overleftrightarrow{XW}$ $\overleftrightarrow{WY}$ $\overleftrightarrow{YW}$ $\overleftrightarrow{XY}$ $\overleftrightarrow{YX}$

b. a plane containing point X

One plane that can be named is plane $\mathcal{P}$. You can also use the letters of any three *noncollinear* points to name this plane.

plane XZY plane VZW plane VZX
plane VZY plane WZX plane WZY

The letters of each of these names can be reordered to create other acceptable names for this plane. For example, XZY can also be written as XYZ, ZXY, ZYX, YXZ, and YZX. In all, there are 36 different three-letter names for this plane.

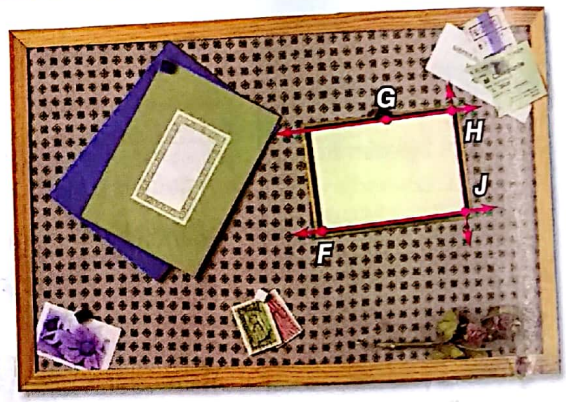
StudyTip
Additional Planes Although not drawn in Example 1b, there is another plane that contains point X. Since points W, T, and X are noncollinear, point X is also in plane WTX.

GuidedPractice

1A. a plane containing points T and Z **1B.** a line containing point T

Real-World Example 2 Model Points, Lines, and Planes

MESSAGE BOARD Name the geometric terms modeled by the objects in the picture.



The push pin models point G.

The maroon border on the card models line GH.

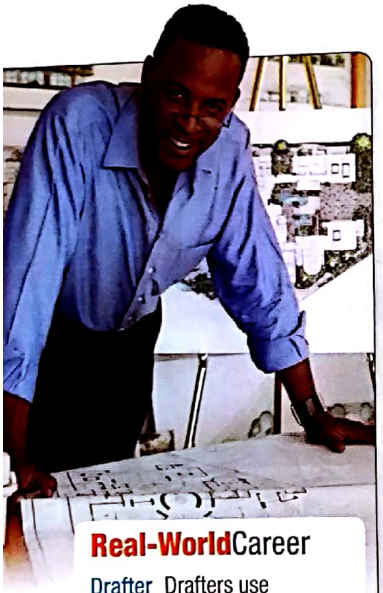
The edge of the card models line HJ.

The card itself models plane FGJ.

GuidedPractice

Name the geometric term modeled by each object.

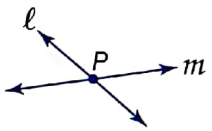
2A. stripes on a sweater **2B.** the corner of a box



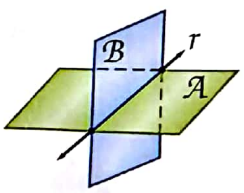
Real-WorldCareer
Drafter Drafters use perspective to create drawings to build everything from toys to school buildings. Drafters need skills in math and computers. They get their education at trade schools, community colleges, and some 4-year colleges. Refer to Exercises 50 and 51.

2 Intersections of Lines and Planes

The **intersection** of two or more geometric figures is the set of points they have in common. Two lines intersect in a point. Lines can intersect planes, and planes can intersect each other.



P represents the intersection of lines ℓ and m.

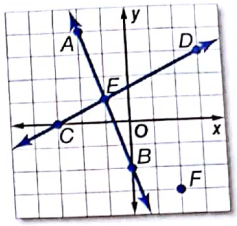


Line r represents the intersection of planes $\mathcal{A}$ and $\mathcal{B}$.

Example 3 Draw Geometric Figures

Draw and label a figure for each relationship.

a. ALGEBRA Lines AB and CD intersect at E for $A(-2, 4)$, $B(0, -2)$, $C(-3, 0)$, and $D(3, 3)$ on a coordinate plane. Point F is coplanar with these points, but not collinear with $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

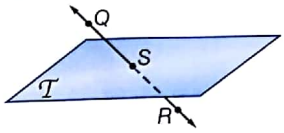


Graph each point and draw $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Label the intersection point as E.

An infinite number of points are coplanar with A, B, C, D and E but not collinear with $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. In the graph, one such point is $F(2, -3)$.

b. QR intersects plane $\mathcal{T}$ at point S.



Draw a surface to represent plane $\mathcal{T}$ and label it.

Draw a dot for point S anywhere on the plane and a dot that is not on plane $\mathcal{T}$ for point Q.

Draw a line through points Q and S. Dash the line to indicate the portion hidden by the plane. Then draw another dot on the line and label it R.

StudyTip
Three-Dimensional Drawings Because it is impossible to show an entire plane in a figure, edged shapes with different shades of color are used to represent planes.

GuidedPractice

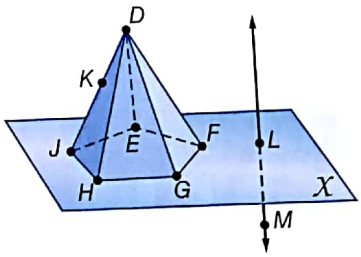
3A. Points $J(-4, 2)$, $K(3, 2)$, and L are collinear.

3B. Line p lies in plane $\mathcal{N}$ and contains point L.

Definitions or **defined terms** are explained using undefined terms and/or other defined terms. **Space** is defined as a boundless, three-dimensional set of all points. Space can contain lines and planes.

Example 4 Interpret Drawings

a. How many planes appear in this figure?



Six: plane X, plane JDH, plane JDE, plane EDF, plane FDG, and plane HDG.

b. Name three points that are collinear.

Points J, K, and D are collinear.

c. Name the intersection of plane HDG with plane X.

Plane HDG intersects plane X in $\overleftrightarrow{HG}$.

b. At what point do $\overleftrightarrow{LM}$ and $\overleftrightarrow{EF}$ intersect? Explain.

It does not appear that these lines intersect. $\overleftrightarrow{EF}$ lies in plane X, but only point L of $\overleftrightarrow{LM}$ lies in X.

StudyTip
Dimension A point has no dimension. A line exists in one dimension. However, a circle is two-dimensional, and a pyramid is three-dimensional.

GuidedPractice

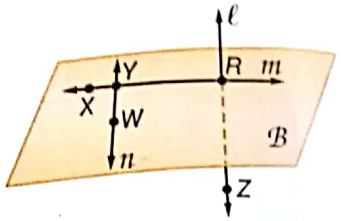
Explain your reasoning.

4A. Are points E, D, F, and G coplanar?

4B. At what point or in what line do planes JDH, JDE, and EDF intersect?

= Step-by-Step Solutions begin on page R20.

Check Your Understanding



Example 1 **Use the figure to name each of the following.**

1. a line containing point X
2. a line containing point Z
3. a plane containing points W and R

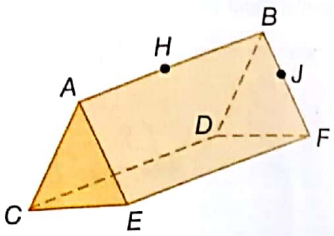
Example 2 **Name the geometric term modeled by each object.**

4. a beam from a laser
5. a floor

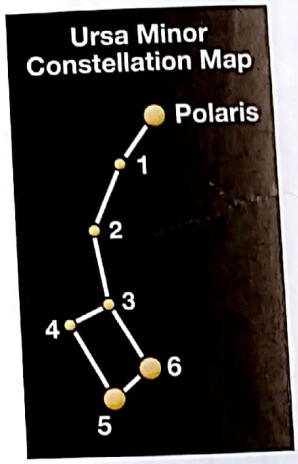
Example 3 **Draw and label a figure for each relationship.**

6. A line in a coordinate plane contains $A(0, -5)$ and $B(3, 1)$ and a point C that is not collinear with $\overleftrightarrow{AB}$.
7. Plane Z contains lines x, y, w. Lines x and y intersect at point V and lines x and w intersect at point P.

Example 4 **Refer to the figure.**



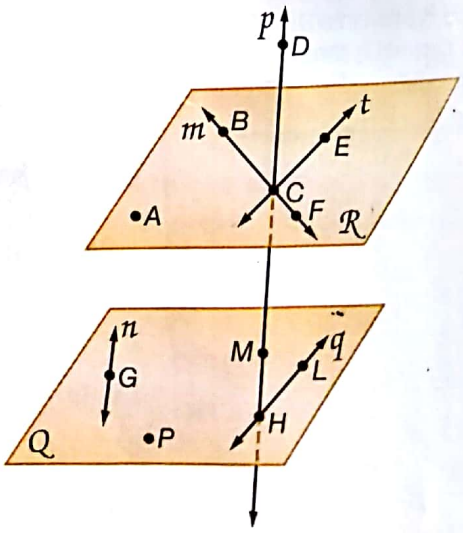
8. How many planes are shown in the figure?
9. Name three points that are collinear.
10. Are points A, H, J, and D coplanar? Explain.
11. Are points B, D, and F coplanar? Explain.
12. **ASTRONOMY** Ursa Minor, or the Little Dipper, is a constellation made up of seven stars in the northern sky including the star Polaris.
    a. What geometric figures are modeled by the stars?
    b. Are Star 1, Star 2, and Star 3 collinear on the constellation map? Explain.
    c. Are Polaris, Star 2, and Star 6 coplanar on the map?



Practice and Problem Solving

Extra Practice begins on page 969.

Example 1 **Refer to the figure.**



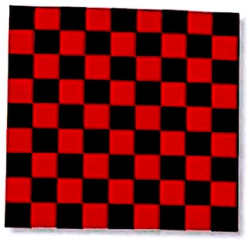
13. Name the lines that are only in plane Q.
14. How many planes are labeled in the figure?
15. Name the plane containing the lines m and t.
16. Name the intersection of lines m and t.
17. Name a point that is not coplanar with points A, B, and C.
18. Are points F, M, G, and P coplanar? Explain.
19. Name the points not contained in a line shown.
20. What is another name for line t?
21. Does line n intersect line q? Explain.

Example 2 Name the geometric term(s) modeled by each object.

22.

23.

24.



25.

26. a blanket

27. a knot in a rope

28. a telephone pole

29. the edge of a desk

30. two connected hallways

31. a partially opened folder

Example 3 **Draw and label a figure for each relationship.**

32. Line m intersects plane $\mathcal{R}$ at a single point.

33. Two planes do not intersect.

34. Points X and Y lie on $\overleftrightarrow{CD}$.

35. Three lines intersect at point J but do not all lie in the same plane.

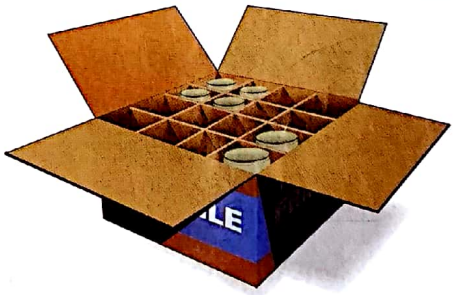
36. Points $A(2, 3)$, $B(2, -3)$, C and D are collinear, but A, B, C, D, and F are not.

37. Lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{NP}$ are coplanar but do not intersect.

38. $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$ intersect at $P(4, 3)$, where point F is at $(-2, 5)$ and point J is at $(7, 9)$.

39. Lines s and t intersect, and line v does not intersect either one.

Example 4 **PACKING When packing breakable objects such as glasses, movers frequently use boxes with inserted dividers like the one shown.**

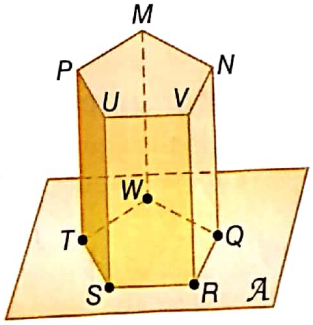


40. How many planes are modeled in the picture?

41. What parts of the box model lines?

42. What parts of the box model points?

Refer to the figure at the right.



43. Name two collinear points.

44. How many planes appear in the figure?

45 Do plane $\mathcal{A}$ and plane MNP intersect? Explain.

46. In what line do planes $\mathcal{A}$ and QRV intersect?

47. Are points T, S, R, Q, and V coplanar? Explain.

48. Are points T, S, R, Q, and W coplanar? Explain.