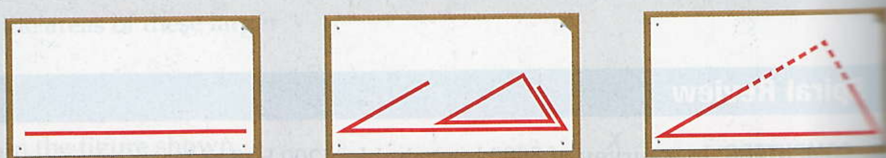
LESSON 7-3

Similar Triangles

:: Then	:: Now	:: Why?
• You used the AAS, SSS, and SAS Congruence Theorems to prove triangles congruent. (Lessons 4-4 and 4-5)	• **1** Identify similar triangles using the AA Similarity Postulate and the SSS and SAS Similarity Theorems. **2** Use similar triangles to solve problems.	• Julian wants to draw a similar version of his skate club's logo on a poster. He first draws a line at the bottom of the poster. Next, he uses a cutout of the original triangle to copy the two bottom angles. Finally, he extends the noncommon sides of the two angles.



Tennessee Curriculum Standards

CLE 3108.4.8 Establish processes for determining congruence and similarity of figures, especially as related to scale factor, contextual applications, and transformations.

✓ 3108.4.36 Use several methods, including AA, SSS, and SAS, to prove that two triangles are similar.

SPI 3108.4.11 Use basic theorems about similar and congruent triangles to solve problems. *Also addresses ✓3108.4.37.*

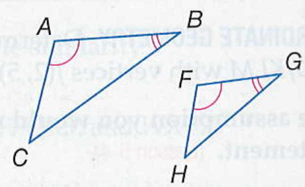
1 Identify Similar Triangles

The example suggests that two triangles are similar if two pairs of corresponding angles are congruent.

Postulate 7.1 Angle-Angle (AA) Similarity

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

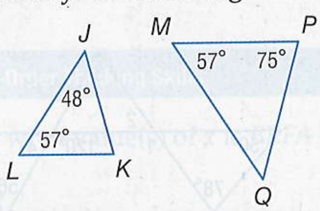
Example If $\angle A \cong \angle F$ and $\angle B \cong \angle G$, then $\triangle ABC \sim \triangle FGH$.



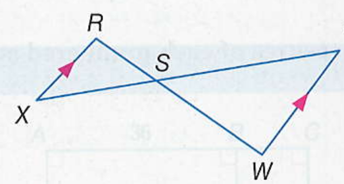
Example 1 Use the AA Similarity Postulate

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

a.



b.

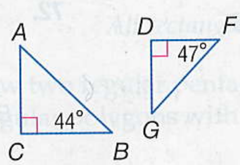


a. Since $m\angle L = m\angle M$, $\angle L \cong \angle M$. By the Triangle Sum Theorem, $57 + 48 + m\angle K = 180$, so $m\angle K = 75$. Since $m\angle P = 75$, $\angle K \cong \angle P$. So, $\triangle LJK \sim \triangle MQP$ by AA Similarity.

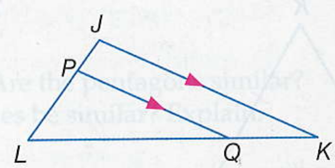
b. $\angle RSX \cong \angle WST$ by the Vertical Angles Theorem. Since $\overline{RX} \parallel \overline{TW}$, $\angle R \cong \angle W$. So, $\triangle RSX \sim \triangle WST$ by AA Similarity.

GuidedPractice

1A.



1B.

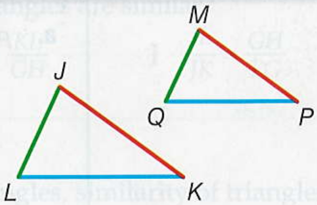


You can use the AA Similarity Postulate to prove the following two theorems.

Theorems Points on Perpendicular Bisectors

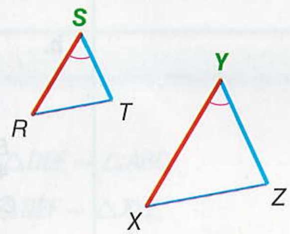
7.2 Side-Side-Side (SSS) Similarity
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

Example If $\frac{JK}{MP} = \frac{KL}{PQ} = \frac{LJ}{QM}$, then $\triangle JKL \sim \triangle MPQ$.



7.3 Side-Angle-Side (SAS) Similarity
If the lengths of two sides of one triangle are proportional to the lengths of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Example If $\frac{RS}{XY} = \frac{ST}{YZ}$ and $\angle S \cong \angle Y$, then $\triangle RST \sim \triangle XYZ$.

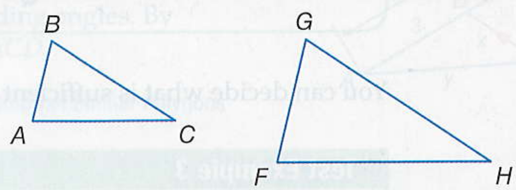


You will prove Theorem 7.3 in Exercise 25.

Proof Theorem 7.2

Given: $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$

Prove: $\triangle ABC \sim \triangle FGH$

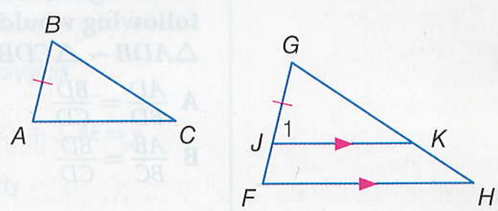


StudyTip

Corresponding Sides To determine which sides of two triangles correspond, begin by comparing the longest sides, then the next longest sides, and finish by comparing the shortest sides.

Paragraph Proof:

Locate J on $\overline{FG}$ so that $JG = AB$. Draw $\overline{JK}$ so that $\overline{JK} \parallel \overline{FH}$. Label $\angle GJK$ as $\angle 1$.



Since $\angle G \cong \angle G$ by the Reflexive Property and $\angle 1 \cong \angle F$ by the Corresponding Angles Postulate, $\triangle GJK \sim \triangle GFH$ by the AA Similarity Postulate.

By the definition of similar polygons, $\frac{JG}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$. By substitution, $\frac{AB}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$.

Since we are also given that $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$, we can say that $\frac{GK}{GH} = \frac{BC}{GH}$ and $\frac{JK}{FH} = \frac{AC}{FH}$. This means that $GK = BC$ and $JK = AC$, so $\overline{GK} \cong \overline{BC}$ and $\overline{JK} \cong \overline{AC}$.

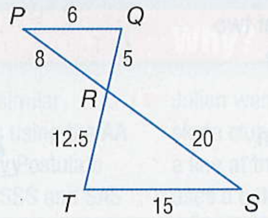
By SSS, $\triangle ABC \cong \triangle JGK$.

By CPCTC, $\angle B \cong \angle G$ and $\angle A \cong \angle 1$. Since $\angle 1 \cong \angle F$, $\angle A \cong \angle F$ by the Transitive Property. By AA Similarity, $\triangle ABC \sim \triangle FGH$.

Example 2 Use the SSS and SAS Similarity Theorems

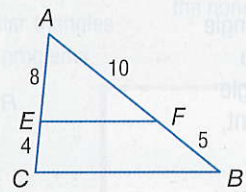
Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

a.



$\frac{PR}{SR} = \frac{8}{20}$ or $\frac{2}{5}$, $\frac{PQ}{ST} = \frac{6}{15}$ or $\frac{2}{5}$, and $\frac{QR}{TR} = \frac{5}{12.5} = \frac{50}{125}$ or $\frac{2}{5}$. So, $\triangle PQR \sim \triangle STR$ by the SSS Similarity Theorem.

b.



By the Reflexive Property, $\angle A \cong \angle A$.

$\frac{AF}{AB} = \frac{10}{10 + 5} = \frac{10}{15}$ or $\frac{2}{3}$ and $\frac{AE}{AC} = \frac{8}{8 + 4} = \frac{8}{12}$ or $\frac{2}{3}$.

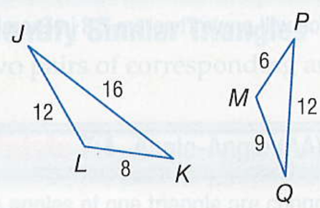
Since the lengths of the sides that include $\angle A$ are proportional, $\triangle AEF \sim \triangle ACB$ by the SAS Similarity Theorem.

StudyTip

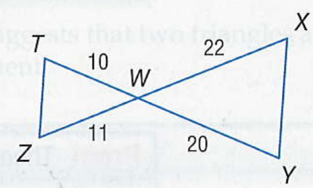
Draw Diagrams It is helpful to redraw similar triangles so that the corresponding side lengths have the same orientation.

GuidedPractice

2A.



2B.

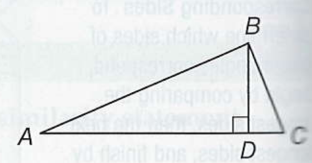


You can decide what is sufficient to prove that two triangles are similar.

SPI 3108.1.4

Test Example 3

In the figure, $\angle ADB$ is a right angle. Which of the following would *not* be sufficient to prove that $\triangle ADB \sim \triangle CDB$?



A $\frac{AD}{BD} = \frac{BD}{CD}$

B $\frac{AB}{BC} = \frac{BD}{CD}$

C $\angle ABD \cong \angle C$

D $\frac{AD}{BD} = \frac{BD}{CD} = \frac{AB}{BC}$

Read the Test Item

You are given that $\angle ADB$ is a right angle and asked to identify which additional information would not be enough to prove that $\triangle ADB \sim \triangle CDB$.

Solve the Test Item

Since $\angle ADB$ is a right angle, $\angle CDB$ is also a right angle. Since all right angles are congruent, $\angle ADB \cong \angle CDB$. Check each answer choice until you find one that does not supply a sufficient additional condition to prove that $\triangle ADB \sim \triangle CDB$.

Choice A: If $\frac{AD}{BD} = \frac{BD}{CD}$ and $\angle ADB \cong \angle CDB$, then $\triangle ADB \sim \triangle CDB$ by SAS Similarity.

Choice B: If $\frac{AB}{BC} = \frac{BD}{CD}$ and $\angle ADB \cong \angle CDB$, then we cannot conclude that $\triangle ADB \sim \triangle CDB$ because the included angle of side $\overline{AB}$ and $\overline{BD}$ is not $\angle ADB$. So the answer is B.

Test-Taking Tip

Identifying Nonexamples Sometimes test questions require you to find a nonexample, as in this case. You must check each option until you find a valid nonexample. If you would like to check your answer, confirm that each additional option is correct.

GuidedPractice

3. If $\triangle JKL$ and $\triangle FGH$ are two triangles such that $\angle J \cong \angle F$, which of the following would be sufficient to prove that the triangles are similar?

F $\frac{KL}{GH} = \frac{JL}{FH}$ G $\frac{JL}{JK} = \frac{FH}{FG}$ H $\frac{JK}{FG} = \frac{KL}{GH}$ J $\frac{JL}{JK} = \frac{GH}{FG}$

2 Use Similar Triangles

Like the congruence of triangles, similarity of triangles is reflexive, symmetric, and transitive.

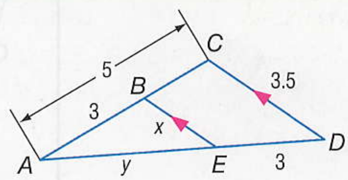
Theorem 7.4 Properties of Similarity

Reflexive Property of Similarity	$\triangle ABC \sim \triangle ABC$
Symmetric Property of Similarity	If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.
Transitive Property of Similarity	If $\triangle ABC \sim \triangle DEF$, and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.

You will prove Theorem 7.4 in Exercise 26.

Example 4 Parts of Similar Triangles

Find BE and AD.



Since $\overline{BE} \parallel \overline{CD}$, $\angle ABE \cong \angle BCD$ and $\angle AEB \cong \angle EDC$ because they are corresponding angles. By AA Similarity, $\triangle ABE \sim \triangle ACD$.

StudyTip

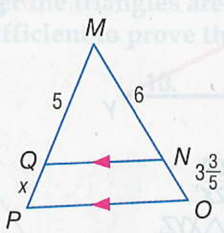
Proportions An additional proportion that is true for Example 4 is $\frac{AC}{CD} = \frac{AB}{BE}$.

$\frac{AB}{AC} = \frac{BE}{CD}$	Definition of Similar Polygons
$\frac{3}{5} = \frac{x}{3.5}$	$AC = 5$, $CD = 3.5$, $AB = 3$, $BE = x$
$3.5 \cdot 3 = 5 \cdot x$	Cross Products Property
$2.1 = x$	BE is 2.1.
$\frac{AC}{AB} = \frac{AD}{AE}$	Definition of Similar Polygons
$\frac{5}{3} = \frac{y+3}{y}$	$AC = 5$, $AB = 3$, $AD = y + 3$, $AE = y$
$5 \cdot y = 3(y + 3)$	Cross Products Property
$5y = 3y + 9$	Distributive Property
$2y = 9$	Subtract $3y$ from each side.
$y = 4.5$	AD is $y + 3$ or 7.5.

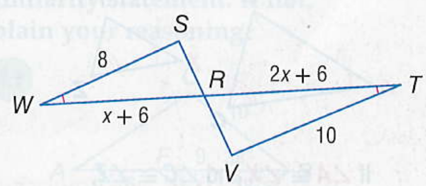
GuidedPractice

Find each measure.

4A. QP and MP



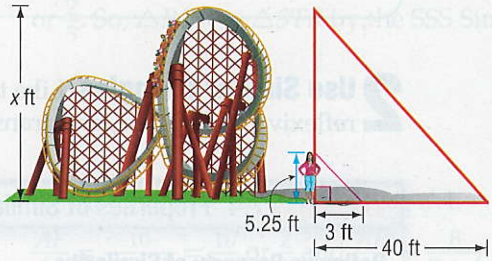
4B. WR and RT



Real-World Example 5 Indirect Measurement

ROLLER COASTERS Hallie is estimating the height of the Superman roller coaster in Mitchellville, Maryland. She is 5 feet 3 inches tall and her shadow is 3 feet long. If the length of the shadow of the roller coaster is 40 feet, how tall is the roller coaster?

Understand Make a sketch of the situation. 5 feet 3 inches is equivalent to 5.25 feet.



Plan In shadow problems, you can assume that the angles formed by the Sun's rays with any two objects are congruent and that the two objects form the sides of two right triangles.

Since two pairs of angles are congruent, the right triangles are similar by the AA Similarity Postulate. So, the following proportion can be written.

$$\frac{\text{Hallie's height}}{\text{coaster's height}} = \frac{\text{Hallie's shadow length}}{\text{coaster's shadow length}}$$

Solve Substitute the known values and let x = roller coaster's height.

$\frac{5.25}{x} = \frac{3}{40}$ Substitution

$3 \cdot x = 40(5.25)$ Cross Products Property

$3x = 210$ Simplify.

$x = 70$ Divide each side by 3.

The roller coaster is 70 feet tall.

Check The roller coaster's shadow length is $\frac{40\text{ ft}}{3\text{ ft}}$ or about 13.3 times Hallie's shadow length. Check to see that the roller coaster's height is about 13.3 times Hallie's height. $\frac{70\text{ ft}}{5.25\text{ ft}} \approx 13.3$ ✓

Problem-Solving Tip

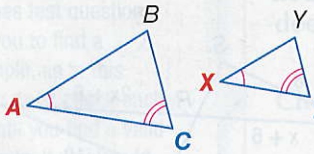
Reasonable Answers When you have solved a problem, check your answer for reasonableness. In this example, Hallie's shadow is a little more than half her height. The coaster's shadow is also a little more than half of the height you calculated. Therefore, the answer is reasonable.

Guided Practice

5. **BUILDINGS** Adam is standing next to the Palmetto Building in Columbia, South Carolina. He is 6 feet tall and the length of his shadow is 9 feet. If the length of the shadow of the building is 322.5 feet, how tall is the building?

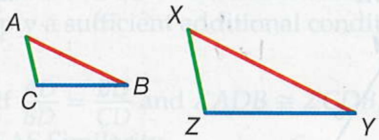
Concept Summary Triangle Similarity

AA Similarity Postulate



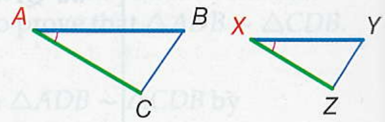
If $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, then $\triangle ABC \sim \triangle XYZ$.

SSS Similarity Theorem



If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

SAS Similarity Theorem



If $\angle A \cong \angle X$ and $\frac{AB}{XY} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.