

Slope and Rate of Change

rate of change = how quickly something is changing in relation to some quantity.

Ex) time, money

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

$$= \frac{y}{x}$$

$$= \frac{\Delta y}{\Delta x}$$

$$\text{slope (m)} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Writing the Equation of a Line (in slope-intercept form)

Recall: the slope-intercept form of an equation is $y = mx + b$

slope \uparrow \uparrow y-intercept

We will use point-slope to write the equation of a line

$$\text{Point-slope} \Rightarrow y - y_1 = m(x - x_1) *$$

point

We need: 1) slope
2) point

Ex) Write the equation of the line through $(-8, 1)$
with slope $m = -\frac{3}{4}$.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -\frac{3}{4}(x - (-8))$$

Given a linear equation in standard form, $Ax + By = C$, write in slope-intercept form.

Ex) $2x + 3y = 5$

Obj: To solve for "y"

$$\begin{array}{r} 2x + 3y = 5 \\ -2x \qquad \qquad -2x \\ \hline \end{array}$$

$$\frac{3y}{3} = \frac{-2x + 5}{3}$$

$$y = -\frac{2}{3}x + \frac{5}{3}$$

Leave as a fraction

Ex) #15 Pg 307

$(5, -8)$; $m = -3$
 x_1 y_1

Use $y - y_1 = m(x - x_1)$

$$y - (-8) = -3(x - 5)$$

$$\begin{array}{r} y + 8 = -3x + 15 \\ -8 \qquad \qquad -8 \end{array}$$

$$y = -3x + 7$$

$$y - 1 = -\frac{3}{4}(x + 8)$$

$$-\frac{3}{4} \cdot \frac{8}{1} = -\frac{24}{4}$$

$$y - 1 = -\frac{3}{4}x - 6$$

+1 +1

$$= -6$$

$$y = -\frac{3}{4}x - 5$$

Ex) Write an equation of the ^{line} ~~line~~ with the given point and slope.

$$(x, y) = \cancel{(-4, 9)} (-4, -9)$$

$$m = 2$$

$$y - y_1 = m(x - x_1)$$

$$y + 9 = 2(x + 4)$$

$$y + 9 = 2x + 8$$

$$\begin{array}{r} -9 \\ -9 \end{array}$$

$$y = 2x - 1$$

$$y - (-9) = 2(x - (-4))$$

Distributive Prop

Solve for "y"

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-1 - 4}{4 - 3} = \frac{-5}{1} = -5$$

$$\textcircled{2} m = \frac{5 - (-7)}{-3 - (-4)} = \frac{12}{1} = 12$$

$$\textcircled{3} m = \frac{5 - 5}{-1 - 2} = \frac{0}{-3} = 0$$

Pg 189 (bottom of pg)

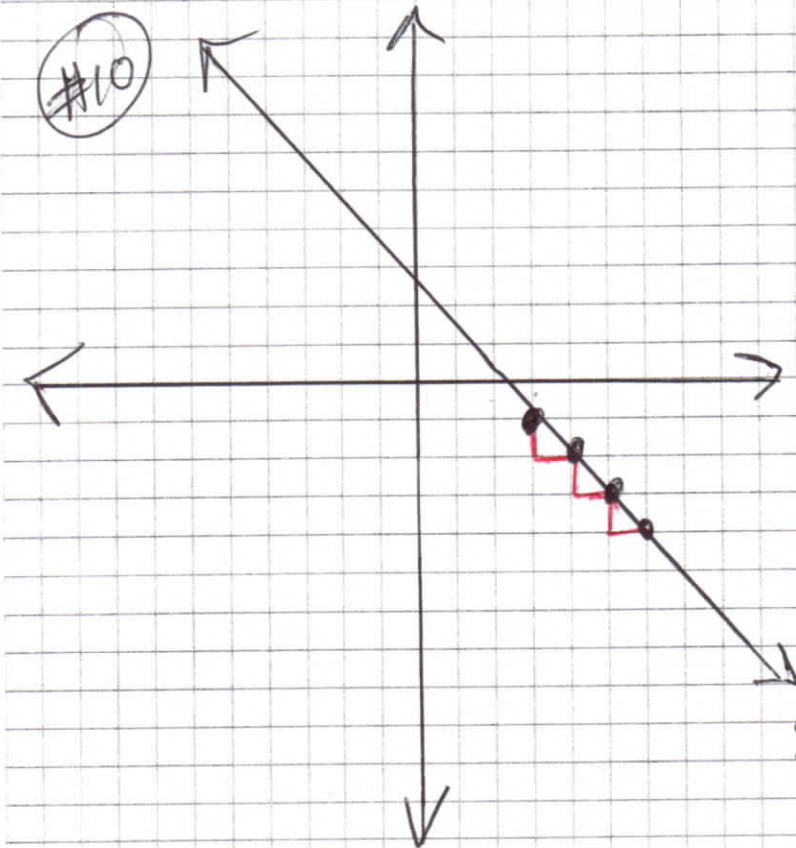
$\textcircled{\# 3A}$ A(14, 13) B(-11, 0) } C(-3, 7) D(-4, -5)

$$m = \frac{0 - 13}{-11 - 14} = \frac{-13}{-25} = \frac{13}{25}$$

$$m = \frac{-5 - 7}{-4 + 3} = \frac{-12}{-1} = 12$$

$\textcircled{\text{Neither}}$

Compare the slopes



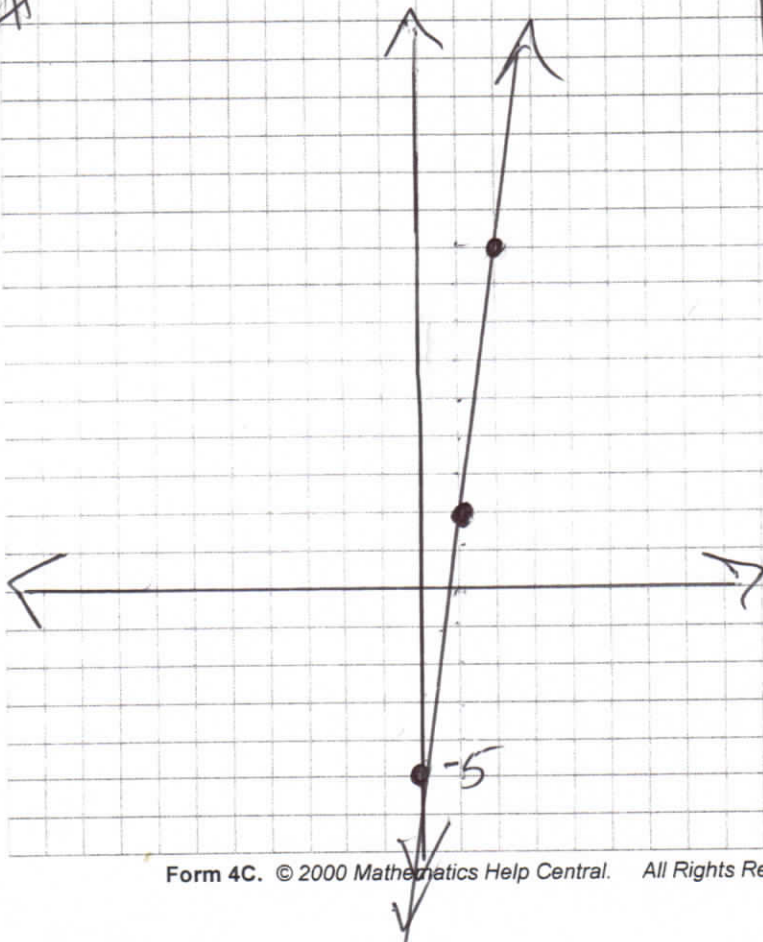
When graphing,
We need 1) slope,
2) a point.

We will graph the
point first and
then the slope.

$$\text{slope} = -1 = \frac{-1}{1}$$

Decreasing slope -
the line goes from
upper left to bottom
right.

#9



$$(x, y) = (1, 2)$$

$$\text{slope} = 7 = \frac{7}{1} = \frac{\text{rise}}{\text{run}}$$

$$y - 2 = 7(x - 1)$$

$$y - 2 = 7x - 7$$

$$+2 \quad +2$$

$$y = 7x - 5$$

Warm Up Pg 202 #50

Know: $m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow$ slope formula

$y - y_1 = m(x - x_1) \Rightarrow$ point-slope formula

$y - 2 = 3(x + 7)$

$y - 2 = 3x + 21$

$+2 \qquad +2$

$y = 3x + 23$
slope

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 4)$

$y - 2 = 3x - 12$

$+2 \qquad +2$

$y = 3x - 10$

$m = \frac{1}{2}$
 $(3, 1)$

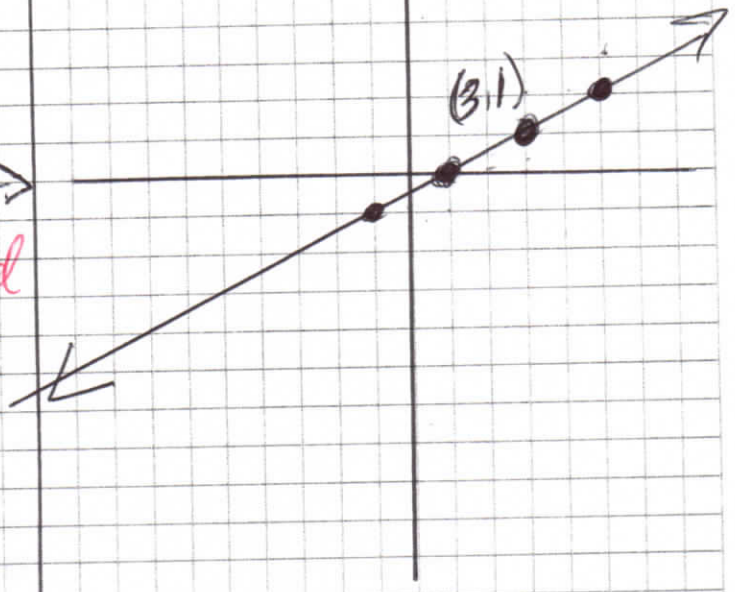
15

#14



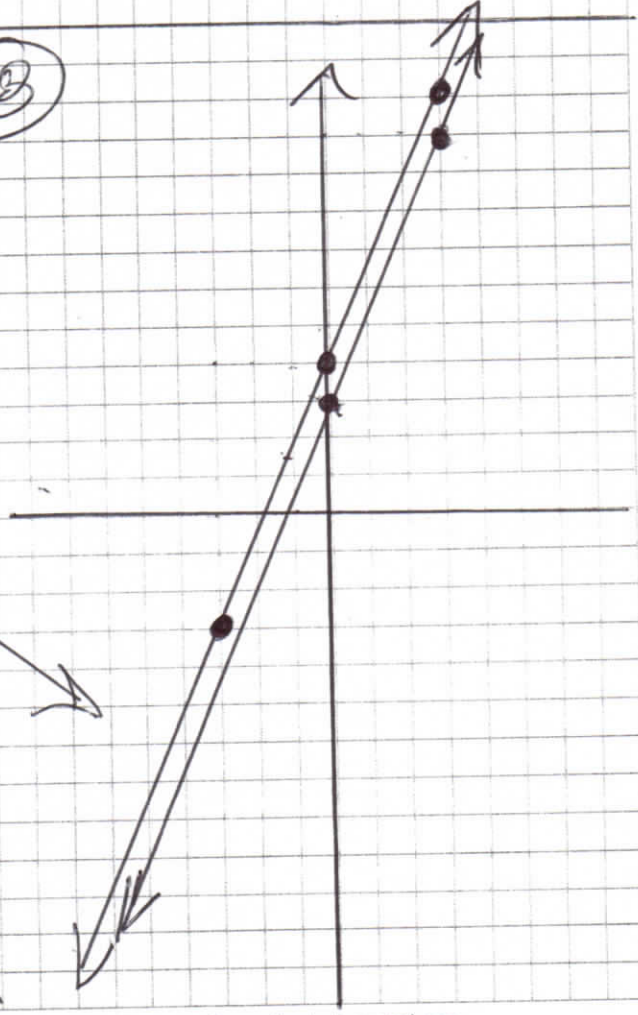
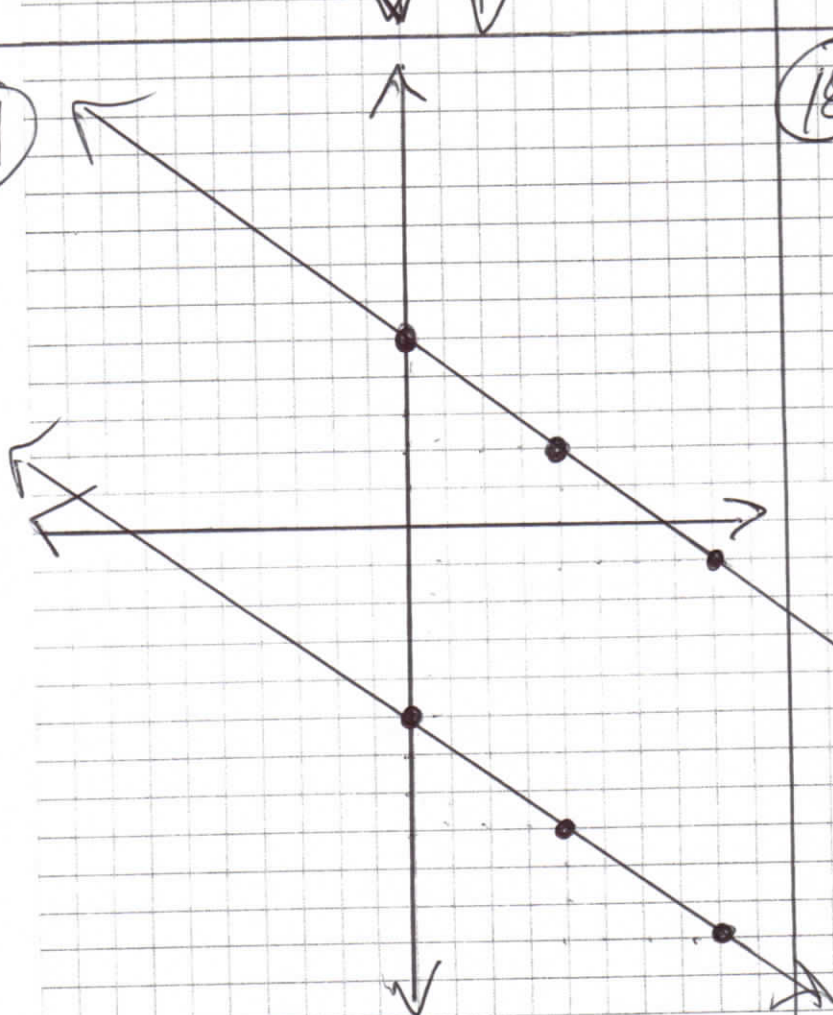
$x = 2$

undefined
 slope
 ↓
 vertical
 line



17

18



To write an equation of a line:

* ① $m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow$ slope formula

② $y - y_1 = m(x - x_1)$

* ③ point

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad (5, -2) \text{ and } (-2, 1)$$

$x_1 \quad y_1 \qquad \qquad x_2 \quad y_2$

$$m = \frac{1 - (-2)}{-2 - 5} = \frac{3}{-7}$$

$$y - 1 = -\frac{3}{7}(x - (-2))$$

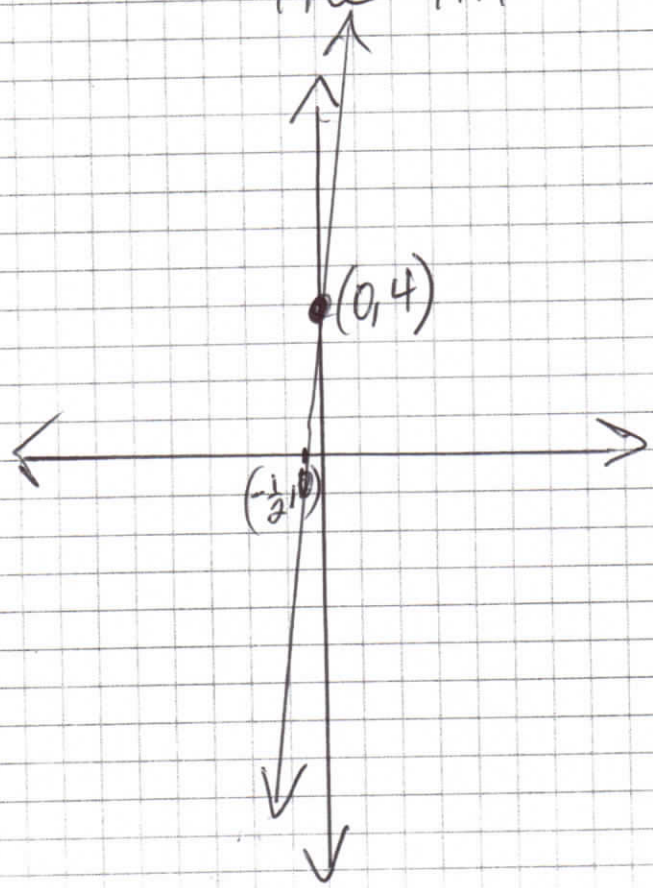
$$y - 1 = -\frac{3}{7}(x + 2)$$

$$y - 1 = -\frac{3}{7}x - \frac{6}{7}$$

$+1 \qquad \qquad +1$

$$y = -\frac{3}{7}x + \frac{1}{7}$$

Ex) Write the equation of the line with x-intercept of $-\frac{1}{2}$ and y-intercept of 4. Graph the line.



x-int $(x, 0)$ y-int $(0, y)$

$(-\frac{1}{2}, 0)$ $(0, 4)$
 x_1 y_1 x_2 y_2

$$m = \frac{4 - 0}{0 - (-\frac{1}{2})} = \frac{4}{\frac{1}{2}} = 4 \cdot \frac{2}{1} = 8$$

$$y - 4 = 8(x - 0)$$

$$y - 4 = 8x$$

$$\quad +4 \qquad +4$$

$$y = 8x + 4$$

Bottom left to upper right \Rightarrow positive slope

Graph 11-18 from Writing Linear Equations Worksheet.

#9 through $(1, 2)$, slope = 7
 x_1, y_1

Use $y - y_1 = m(x - x_1)$

$$y - 2 = 7(x - 1)$$

$$y - 2 = 7x - 7$$

$$y = 7x - 5$$

slope
(this slope is increasing)

y-intercept.
(where the line cross (intercepts) the y-axis)
 $(0, y)$

Write the equation of the line
through $\left(\frac{1}{2}, \frac{3}{2}\right)$ and $\left(-\frac{1}{4}, \frac{5}{4}\right)$.

x_1 y_1 x_2 y_2

$$m = \frac{\frac{5}{4} - \frac{3}{2}}{-\frac{1}{4} - \frac{1}{2}} = \frac{-\frac{1}{4}}{-\frac{3}{4}} = -\frac{1}{4} \cdot \left(-\frac{4}{3}\right) = \frac{1}{3}$$

↑
slope

$$\frac{5}{4} = \frac{5}{4}$$

$$\frac{3}{2} = \frac{6}{4}$$

$$-\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2} = \frac{1}{3}\left(x - \frac{1}{2}\right)$$

$$y - \frac{3}{2} = \frac{1}{3}x - \frac{1}{6}$$

$$+\frac{3}{2}$$

$$+\frac{3}{2}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

Write the equation of the line
through $(-\frac{2}{3}, \frac{8}{3})$ and $(\frac{1}{3}, \frac{7}{3})$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{7}{3} - \frac{8}{3}}{\frac{1}{3} - (-\frac{2}{3})} = \frac{-\frac{1}{3}}{1} = \left(-\frac{1}{3}\right)$$

$$y - y_1 = m(x - x_1) \quad \text{point-slope}$$

$$y - \frac{7}{3} = -\frac{1}{3}\left(x - \frac{1}{3}\right)$$

$$y - \frac{7}{3} = -\frac{1}{3}x + \frac{1}{9} = \frac{1}{9}$$

$$+\frac{7}{3} \qquad \frac{7}{3} = \frac{21}{9}$$

$$y = -\frac{1}{3}x + \frac{22}{9}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{3}{7}(x - 5)$$

$$y + 2 = -\frac{3}{7}x + \frac{15}{7} = \frac{15}{7}$$

$$-2 \qquad -2 = -\frac{14}{7}$$

$$y = -\frac{3}{7}x + \frac{1}{7}$$

$$\frac{15}{7} = \frac{15}{7} = \frac{15}{7}$$

$$\ominus 2 = \frac{2}{1} = \frac{14}{7}$$
$$\frac{1}{7}$$

Ex) Write the equation of the line passing through $(9, 4)$, $(17, 6)$.

$$m = \frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{4}(x - 9)$$

$$y - 4 = \frac{1}{4}x - \frac{9}{4}$$

$$\begin{array}{r} +4 \qquad \qquad +4 = \frac{16}{4} \\ \hline \end{array}$$

$$y = \frac{1}{4}x + \frac{7}{4}$$

$$-\frac{9}{4} = -\frac{9}{4}$$

$$+\frac{4}{1} = \frac{16}{4}$$

$$= \frac{7}{4}$$

Pg 287 (10-20) evens
Pg 307-308 (10-~~30~~) evens;

Write the equation of the line through the points:

a) $(-2, 1)$ and $(3, 11)$

b) $(-\frac{4}{5}, \frac{1}{8})$ and $(-\frac{8}{5}, -\frac{3}{8})$

c) $(-2, -4)$ and $(-2, 7)$

* d) Write the equation of the line through $(9, -3)$ parallel to $2x + 3y = -12$



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

Ex) ~~Ex)~~ Find the slope between the points $A(4, -1)$ and $B(3, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(4 - (-1))}{3 - 4} = \frac{5}{-1} = \textcircled{-5}$$

Ex) Find the slope between the points between $M(3, 7)$ and $N(-5, -9)$

$$m = \frac{-9 - 7}{-5 - 3} = \frac{-16}{-8} = \textcircled{2}$$

Pg 202 #52

$$y + 11 = \frac{1}{2}(4x + 6)$$

$$y + 11 = 2x + 3$$

$$\begin{array}{r} -11 \\ -11 \end{array}$$

$$y = 2x - 8$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 2(x - 5)$$

$$y - 3 = 2x - 10$$

$$\begin{array}{r} +3 \\ +3 \end{array}$$

$$y = 2x - 7$$

Warmup Pg 202 #52

$$y + 11 = \frac{1}{2}(4x + 6)$$

$$y + 11 = 2x + 3$$

$$\begin{array}{r} -11 \\ \hline y = 2x - 8 \end{array}$$

↑ slope

(5, 3)

So, $y - 3 = 2(x - 5)$

$$y - 3 = 2x - 10$$

$$\begin{array}{r} + 3 \\ \hline y = 2x - 7 \end{array}$$

* Parallel lines have the same slope.

#23

through $(-2, 4)$; parallel to

$$y = -\frac{5}{2}x + 5$$

$$y - 4 = -\frac{5}{2}(x - (-2))$$

$$y - 4 = -\frac{5}{2}x - 5$$

+4 +4

$$y = -\frac{5}{2}x - 1$$

(17) through $(4, 2)$ parallel to
 $y = -\frac{3}{4}x - 5$

* Parallel lines have the SAME slope.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{4}(x - 4)$$

$$y - 2 = -\frac{3}{4}x + 3$$

$$\begin{array}{r} +2 \\ +2 \end{array}$$

$$y = -\frac{3}{4}x + 5$$

$(2, 0)$, parallel to $y = \frac{1}{3}x + 3$
 x_1 y_1

#21

Use $y - y_1 = m(x - x_1)$
point-slope

$$y - 0 = \frac{1}{3}(x - 2) \quad \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$$

$$\boxed{y = \frac{1}{3}x - \frac{2}{3}} \quad \text{slope-intercept}$$

#22

$(4, -4)$, parallel to $y = -\frac{1}{1}x - 4$

$$y - (-4) = -1(x - 4)$$

$$y + 4 = -1x + 4$$

$$\begin{array}{r} -4 \qquad \qquad -4 \\ \hline \end{array}$$

$$\boxed{y = -1x}$$